Timothy M. Hsu

Curriculum Vitae

Educational History

6/90	S.B., Mathematics, Mass. Inst. of Technology
9/90	S.B., Music, M.I.T.
11/94	Ph.D., Mathematics, Princeton Univ.

Professional Experience

9/94–6/95	Princeton Univ., Lecturer; organizer, Rutgers-Princeton group theory seminar
9/95–6/98	U. Mich. Ann Arbor, Hildebrandt Res. Asst. Prof.
7/96	MSRI Summer graduate workshop, Mentor
9/97–12/97	U. Mich., Co-coordinator, Math 115 (Calculus I)
9/98–6/01	Pomona College, Visiting Asst. Prof.
8/01–8/06	San José State Univ., Asst. Prof.
8/01–6/12	Director, Center for Applied Mathematics, Computation and Statistics (CAMCOS), SJSU
8/06–8/12	San José State Univ., Assoc. Prof.
8/07–12/07	Member, Mathematical Sciences Research Institute (MSRI) program in Geometric Group Theory
8/12–present	San José State Univ., Prof.
8/13–present	San José State Univ., Coordinator, Math/Stats TA program
8/15–present	San José State Univ., Assoc. Chair, Math/Stats

Awards

5/90	Phi Beta Kappa, M.I.T.
9/90–8/94	NSF Graduate Fellowship
6/96–8/96	Rackham Summer Faculty Fellowship, U. Mich.
9/97	Krasny Prize for outstanding work in motivating undergraduate students, Math Dept., U. Mich.
6/04	Master's student J. Kittock awarded 2004 University Outstanding Thesis Award (two awarded SJSU-wide)
6/08	Master's student K. Shelley Nolan awarded 2008 University Outstanding Thesis Award (two awarded SJSU-wide)
1/12	Intel Science Talent Search advisee C. Day named 2012 semifinalist

Grants Awarded at SJSU

Fall 2001	SJSU Professional Development award, $1,500 for travel
Spring 2002	CAMCOS awarded $43,500 from Woodward Fund, for two semester projects with NASA Ames Research Center

2002–2003	CAMCOS awarded approx. $62,000 from Woodward Fund, for two year-long projects with NASA Ames Research Center
Summer 2002	SJSU summer faculty fellowship, one month summer salary
Fall 2002	SJSU Professional Development award, $1,175 for travel
Spring 2003	LPP planning grant: $2,285 for .1 release time
2003–2004	CAMCOS awarded $61,510 from Woodward Fund, for two year-long projects with NASA Ames Research Center
Fall 2003	SJSU Professional Development award, $1,000 for travel
Fall 2003	LPP implementation grant: $5,000 for .2 release time
Fall 2003	CAMCOS awarded $18,000 from NASA (Macready) for semester project with NASA Ames Research Center
Summer 2004	CAMCOS awarded $4,000 from Woodward Fund, for development of potential CAMCOS project with Numerical Algorithms Group (UK)
2004–2005	CAMCOS awarded $61,510 from Woodward Fund, for two year-long projects with NASA Ames Research Center
Fall 2004	SJSU Professional Development award, $1,000 for travel
Fall 2004	Junior Faculty Career Devlopment Grant: .2 release time
Spring 2005	$2,000 donation from Google to support Bay Area Discrete Math Day conference held at SJSU, 4/9/05
2005–2006	CAMCOS awarded $63,190 from Woodward Fund, for two year-long projects
2005–2006	Sally Casanova Pre-Doctoral $3,000 Scholarship awarded to student Jing-Wei Huang; up to $1,000 goes to faculty sponsor (Hsu) travel expenses
Fall 2005	CAMCOS awarded $15,796 from NASA Ames Research Center for “Intelligent Instruments on Robotic Helicopters”
Spring 2006	CAMCOS awarded $19,330 from Intel Corporation for “Analysis of Heat Pipe Performance Tailored for MEROM/Santa Rosa in Mobile Computers”
2006–2007	CAMCOS awarded $62,600 from Woodward Fund, for two year-long projects with NASA Ames Research Center
2007–2008	CAMCOS awarded $62,600 from Woodward Fund, for two year-long projects with NASA Ames Research Center
2008–2009	CAMCOS awarded $62,000 from Woodward Fund, for two year-long projects with NASA Ames Research Center
2009–2010	CAMCOS awarded $36,500 from Woodward Fund, for one semester-long project with NASA Ames Research

	Center
Spring 2010	University Planning Council Student Success Grant: .2 release time
Spring 2010	(w/ Dr. Maria Cayco) $2,750 awarded from Mathematical Association of America to support Northern California Undergraduate Mathematics Conference
Fall 2010	CAMCOS awarded $13,000 from Woodward Fund, for one semester-long project with NASA Ames Research Center
Spring 2010	CAMCOS awarded $20,400 from Woodward Fund, for one semester-long project with NASA Ames Research Center
Fall 2011	CAMCOS awarded $20,400 from Woodward Fund, for one semester-long project with NASA Ames Research Center
Spring 2013	$2,000 donation from D.E. Shaw to support Bay Area Discrete Math Day conference held at SJSU, 4/6/13
Summer 2013	Undergraduate Research Grant awarded for work with Charles Petersen
Fall 2013	SJSU Research, Scholarship, & Creative Activity Award: .2 release time
Spring 2016–present	Senior personnel, “First in the World” grant for teaching flipped calculus I: .2 release time in multiple semesters

POST-GRADUATE SCHOOL TEACHING EXPERIENCE

Fall 1995	Calculus I (2 sections)
Winter 1996	Transformation groups and geometry
Fall 1996	Calculus II (2 sections)
Winter 1997	Applied modern algebra
1996–1997	Advisor, S. Molnar’s senior thesis in math and creative writing (Virginia Voss award)
Summer 1997	Mentor, summer graduate workshop, MSRI
Fall 1997	Calculus I (also course co-coordinator)
Winter 1998	Introduction to linear algebra
Summer 1998	Codes, ciphers and secret messages, Mich. Math Scholars (mathematically talented high school students)
Fall 1998	Calculus I (2 sections); multivariable calculus
Spring 1999	Multivariable calculus; algebra I
1998–1999	Advisor, R. Derby-Talbot’s senior thesis (honors)
Fall 1999	Calculus I; multivariable calculus; linear algebra
Spring 2000	Calculus II; linear algebra
1999–2000	Advisor, senior theses of A. Draganova (honors), R. Huston, and C. Meyers (honors)

Fall 2000	Calculus I; multivariable calculus; linear algebra
Spring 2001	Alternative calculus II; hyperbolic geometry
2000–2001	Advisor, senior theses of M. Dickerson, J. Singer (honors), and E. Zupunski
Fall 2001	Calculus I; Linear algebra
Spring 2002	Linear algebra; Abstract algebra I
2001–2002	Advisor, master's thesis of A. Vu
Fall 2002	Mathematics for general education (2 sections); Introduction to combinatorics; Reading course on Galois theory (J. Kittock)
Spring 2003–Spring 2004	Advisor, master's thesis of J. Kittock (university honors)
Spring 2003	Mathematics for general education; Linear algebra II
Fall 2003	Mathematics for general education; Linear algebra II
Spring 2004	Mathematics for general education; Introduction to number theory
Fall 2004–Spring 2007	Advisor, master's thesis of P. Darafshi
Fall 2004	Calculus I; Linear algebra II
Spring 2005	Introduction to analysis (2 sections)
Fall 2005–	Advisor, master's thesis of P. Friedenbach
Fall 2005	Calculus III; Abstract algebra I
Spring 2006–Spring 2007	Advisor, master's thesis of M. Bandari
Spring 2006	Calculus II; Vector calculus
Fall 2006–Spring 2007	Advisor, master's thesis of K. Shelley Nolan (university honors)
Fall 2006	Calculus III; Introduction to proof
Spring 2007	Calculus II; Introduction to proof
Spring 2008	Precalculus; Introduction to proof
Fall 2008–Spring 2009	Advisor, master's thesis of S. Dharia
Fall 2008–Spring 2010	Advisor, master's thesis of N. Vazquez
Fall 2008	Precalculus; Introduction to analysis
Spring 2009	Abstract algebra I
Fall 2009–Spring 2011	Advisor, master's thesis of P. Hansen
Fall 2009	Precalculus; Introduction to number theory
Spring 2010	Calculus II
Fall 2010	Precalculus; Abstract algebra I
Spring 2011	Abstract algebra II
Fall 2011–Spring 2013	Advisor, master's thesis of D. Adams
Fall 2011	Precalculus; Introduction to proof
Spring 2012	Linear algebra II; CAMCOS project in applied mathematics
Fall 2012	Precalculus; Introduction to proof

Spring 2013	Precalculus; Introduction to analysis
Fall 2013	Analysis II (Hilbert spaces and applications)
Spring 2014–Spring 2015	Advisor, master's thesis of O. Zamoroueva
Spring 2014	Discrete math; Introduction to proof
Fall 2014–Spring 2016	Advisor, writing project of N. Mittal
Fall 2014	Discrete math; Euclidean geometry
Spring 2015–present	Advisor, master's thesis of C. Parayil
Spring 2015	Introduction to number theory; introduction to analysis
Fall 2015	Calculus III; Analysis II
Spring 2016	Introduction to proof
Fall 2016	Calculus I (flipped); Analysis II
Fall 2017	Calculus I (flipped); Analysis II

Outreach and related activities

Summer 2011	Advised Intel Science Talent Search project of Cynthia Day (Lynbrook High School), *Time complexity and algorithms for Blue-Red CHOMP and its subgames*; project made semifinal round

Conferences and Sessions Co-organized

Fall 2004–	Bay Area Discrete Math Day (bi-annual local conference)
Spring 2005	BAD Math Day at SJSU, local organizer
Summer 2005	MAXENT 2005 (25th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering)
Summer 2007	MAA Mathfest: Panel discussion on "Starting and maintaining a student industrial research progam in the mathematical sciences"
Summer 2007	MAA Mathfest: Contributed paper session on "Student Research in Industrial Mathematics"
Spring 2008	AMS Western Section Meeting: Special session on "Combinatorics of partially ordered sets"
Spring 2010	Northern California Undergraduate Mathematics Conference
Spring 2013	BAD Math Day at SJSU, local organizer

Recent Talks

03/07/12	SJSU Math Colloquium, "Square-gluing puzzles and the Gauss-Bonnet Theorem"
05/04/12	U. Michigan RTG Workshop on Recent Progress on Hyperbolic 3-Manifolds, Ann Arbor, MI: Mini-course lectures "Special Cube Complexes", "Finiteness conditions of the dual cube complex", "Cubulating Malnormal Amalgams"
05/29/13	Cube complexes and 3-manifolds conference, Univ. of Ill.

	at Chicago, Chicago, IL: “Computing the ℓ^2-homology of clean complexes”
04/29/14	Claremont Colleges Math Colloquium, Harvey Mudd Coll., Claremont, CA: “Cube Complexes, 3-manifolds, and the Virtual Haken Theorem”
Fall 2014	Combinatorics Seminar, SJSU, San José, CA: “Introduction to combinatorial game theory” (3-talk series)
10/28/14	Santa Clara Univ. Math Colloquium, Santa Clara Univ., Santa Clara, CA: “Cube complexes, 3-manifolds, and the virtually fibered theorem”
04/15/15	SJSU Math Colloquium, SJSU, San José, CA: “Cube complexes, 3-manifolds, and the Virtually Fibered Conjecture”
04/12/17	Cal Poly Pomona Math/Stats Colloquium, Cal Poly Pomona, Pomona, CA: “How I flip calculus”
04/13/17	CSULA Math Seminar, CSULA, Los Angeles, CA: “How I flip calculus”

Research Interests

Geometric group theory; combinatorics of partially ordered sets; ℓ^2 invariants; combinatorial game theory; finite groups and related topics; cell complexes and low-dimensional topology; loops and quasigroups; computational group theory; undergraduate mathematics education.

Professional Societies

Member of the AMS, MAA, and SIAM.

Computer Skills

Fluent in LaTeX and HTML. Prior experience with C, FORTRAN, GAP, Java, LISP, Maple, Mathematica, MATLAB, Perl, and UNIX. Some professional programming and technical support experience.

Contact Information

E-mail: `tim.hsu@sjsu.edu`
Address: Math Dept., San José State University, San José, CA 95192-0103
Phone: (408)924-5071 (office), (408)924-5080 (fax)
Home Page: `http://www.math.sjsu.edu/~hsu`

Publications (by subject, in reverse chronological order, with selected abstracts)

Geometric group theory

[1] (with E. Berkove, S. Dharia, and R. McGuigan) *The ℓ^2-cohomology of clean complexes*, preprint, 2016.

A *clean complex* is a graph of spaces whose vertex and edge spaces are graphs and whose attaching maps are embeddings. Clean complexes are notable because they provide a 2-dimensional test case for *virtually special* cube complexes. Given an oracle for computing in $U(F_n)$, we present an algorithm for calculating the ℓ^2-homology of many clean complexes, including all *rosy complexes* (clean complexes whose underlying graphs and vertex spaces are n-leave roses). As a result, we obtain large classes of examples whose Betti numbers do not vanish and do not depend only on the Euler characteristic.

[2] (with D. T. Wise) *Cubulating malnormal amalgams*, Invent. Math. **199** (2015), no. 2, 293–331.

In this paper we examine sufficient conditions on a group G splitting as a graph of groups that ensure that G acts properly on a CAT(0) cube complex. The precise conditions are somewhat technical, but they include hyperbolicity of G relative to abelian subgroups, quasiconvexity of the vertex groups, and malnormality and subgroup separability properties of the edge groups. An additional condition involves an extension property for codimension-1 subgroups. Many of the conditions are satisfied automatically when the edge groups are free, or maximal cyclic.

[3] (with D. T. Wise) *Cubulating graphs of free groups with cyclic edge groups*, Amer. J. Math. **132** (2010), no. 5, 1153–1188.

We prove that if G is a finitely generated group that splits as a graph of free groups with cyclic edge groups, and G has no non-Euclidean Baumslag-Solitar subgroups, then G is the fundamental group of a compact nonpositively curved cube complex. In addition, if G is also word-hyperbolic (i.e., if G contains no Baumslag-Solitar subgroups of any type), we show that G is linear (in fact, is a subgroup of $SL_n(Z)$).

[4] (with I. J. Leary) *Artin HNN-extensions virtually embed in Artin groups*, Bull. Lon. Math. Soc. **40** (2008), no. 4, 715–719.

An Artin HNN-extension is an HNN-extension of an Artin group in which the stable letter conjugates a pair of suitably chosen subsets of the standard generating set. We show that some finite index subgroup of an Artin HNN-extension embeds in an Artin group. We also obtain an analogous result for Coxeter groups.

[5] (with D. T. Wise) *Groups with infinitely many types of fixed subgroups*, Israel J. Math. **144** (2004), 93–107.

It is a theorem of Shor that if G is a word-hyperbolic group, then up to isomorphism, only finitely many groups appear as fixed subgroups of automorphisms of G. We give an example of a group G acting freely and cocompactly on a CAT(0) square complex such that infinitely many non-isomorphic groups appear as fixed subgroups of automorphisms of G. Consequently, Shor's finiteness result does not hold if the negative curvature condition is relaxed to either biautomaticity or nonpositive curvature.

[6] (with D. T. Wise) *Ascending HNN extensions of polycyclic groups are residually finite*, J. Pure Appl. Alg. **182** (2003), no. 1, 65–78.

We prove that every ascending HNN extension of a polycyclic-by-finite group is residually finite. We also give a criterion for the residual finiteness of an ascending HNN extension of a residually nilpotent group, and apply this criterion to recover a result of Sapir on the residual finiteness of certain ascending HNN extensions of free groups.

[7] (with D. T. Wise) *Separating quasiconvex subgroups of right-angled Artin groups*, Math. Z. **240** (2002), no. 3, 521–548.

A *graph group*, or *right-angled Artin group*, is a group given by a presentation where the only relators are commutators of the generators. A graph group presentation corresponds in a natural way to a simplicial graph, with each generator corresponding to a vertex, and each commutator relator corresponding to an edge. We show that if H is a quasiconvex subgroup of a right-angled Artin group G whose corresponding graph is a tree, then H is a *separable* subgroup of G, or in other words, H is the intersection of finite index subgroups of G. We also discuss some consequences relating to knot groups and 3-manifolds.

[8] (with D. T. Wise) *On linear and residual properties of graph products*, Mich. Math. J. **46** (1999), 251–259.

If Γ is a graph, and G_v is a set of groups indexed by the vertices of Γ, then the *graph product* $G\Gamma$ is defined to be the free product of the G_v, modulo the relations $[g_v, g_w] = 1$ for all g_v in G_v and g_w in G_w such that (v, w) is an edge in Γ. We show that the graph product of subgroups of Coxeter groups is a subgroup of a Coxeter group. As a result, we obtain short proofs that graph groups (right-angled Artin groups) are linear and that the graph product of residually finite groups is residually finite. We also give a new and more geometric proof of the normal form theorem for graph products.

[9] (with D. T. Wise) *A non-residually finite square of finite groups*, in C. M. Campbell et al. (eds.), *Groups St. Andrews 1997 in Bath, I*, volume 260 of *LMS Lect. Notes*, 368–378. Cambridge Univ. Press, 1999.

We construct a non-positively curved non-residually finite square of finite groups whose vertex groups are of order 288, 288, 576, and 576. (In contrast, the example constructed in [10] has vertex groups of order between 2^{60} and 2^{150}.) In doing so, we demonstrate a new, more geometric method for embedding the fundamental group of a complete squared complex in the fundamental group of a square of finite groups.

[10] (with D. T. Wise) *Embedding theorems for non-positively curved polygons of finite groups*, J. Pure Appl. Alg. **123** (1998), 201–221.

The fundamental groups of complete squared complexes are a class of groups, some of which are not residually finite. A method is given for embedding the fundamental group of a complete squared complex as a subgroup of a square of finite groups, all of whose (Gersten-Stallings) vertex angles are $\leq \pi/2$. It is also shown that every square of finite groups, all of whose vertex angles are $\leq \pi/2$ can be embedded in a non-positively curved triangle of finite groups. In this way, a non-residually finite, non-positively curved triangle of finite groups is obtained.

Combinatorics of partially ordered sets

[11] (with C. Petersen) *Upset-downset*, In preparation, 2017.

[12] (with C. Day) *Blue-red CHOMP*, In preparation, 2017.

[13] (with M. J. Logan and S. Shahriari) *Methods for nesting rank 3 normalized matching rank-unimodal posets*, Disc. Math. **309** (2009), no. 3, 521–531.

Anderson and Griggs proved independently that a rank-symmetric-unimodal normalized matching (NM) poset possesses a nested chain decomposition (or *nesting*), and Griggs later conjectured that this result still holds if we remove the condition of rank-symmetry. We give several methods for constructing nestings of rank-unimodal NM posets of rank 3, which together produce substantial progress towards the rank 3 case of the Griggs nesting conjecture. In particular, we show that certain nearly symmetric posets are nested; we show that certain highly asymmetric rank 3 NM posets are nested; and we use results on minimal rank 1 NM posets to show that certain other rank 3 NM posets are nested.

[14] (with M. J. Logan and S. Shahriari) *The generalized Füredi conjecture holds for finite linear lattices*, Disc. Math. **306** (2006), 3140–3144.

We define an *Anderson-Griggs* poset to be a finite rank-unimodal, rank-symmetric, normalized matching poset P, and we say that P has *rapidly decreasing rank numbers* if below the largest ranks in the middle of P, the size of each level is at most half of the previous one. Generalizing a question of Füredi about the Boolean lattice, we conjecture that every Anderson-Griggs poset of width w has a partition into w chains such that the size of each chain is one of two consecutive integers. We prove the conjecture for all Anderson-Griggs posets with rank ≤ 3, and consequently obtain the conjecture for Anderson-Griggs posets with rapidly decreasing rank numbers. In particular, there exists a partition of the linear lattice $L_n(q)$ (subspaces of an n-dimensional vector space over a field of order q, ordered by inclusion) into chains such that the number of chains is the number of subspaces of dimension $\lfloor n/2 \rfloor$, and the size of each chain is one of two consecutive integers.

[15] (with M. J. Logan, S. Shahriari, and C. Towse) *Partitioning the Boolean lattice into a minimal number of chains of relatively uniform size*, Eur. J. Comb. **24** (2003), no. 2, 219–228.

Let $\mathbf{2}^{[n]}$ denote the *Boolean lattice* of order n, that is, the poset of subsets of $\{1, \ldots, n\}$ ordered by inclusion. Extending our previous work on a question of Füredi, we show that for any $c > 1$, there exist functions $e(n) \sim \sqrt{n}/2$ and $f(n) \sim c\sqrt{n \log n}$ and an integer N (depending only on c) such that for all $n > N$, there is a chain decomposition of the Boolean lattice $\mathbf{2}^{[n]}$ into $\binom{n}{\lfloor n/2 \rfloor}$ chains, all of which have size between $e(n)$ and $f(n)$. (A positive answer to Füredi's question would imply that the same result holds for some $e(n) \sim \sqrt{\pi/2}\sqrt{n}$ and $f(n) = e(n) + 1$.) The main tool used is a matching property of normalized matching posets that is not hard to prove, but does not seem to have been widely used before.

[16] (with M. J. Logan, S. Shahriari, and C. Towse) *Partitioning the Boolean lattice into chains of large minimum size*, J. Comb. Thy. (A) **97** (2002), no. 1, 62–84.

Let $\mathbf{2}^{[n]}$ denote the *Boolean lattice* of order n, that is, the poset of subsets of $\{1, \ldots, n\}$ ordered by inclusion. Recall that $\mathbf{2}^{[n]}$ may be partitioned into what we call the *canonical symmetric chain decomposition* (due to de Bruijn, Tengbergen, and Kruyswijk), or CSCD. Motivated by a question of Füredi, we show that there exists a function $d(n) \sim (1/2)\sqrt{n}$ such that for any $n \geq 0$, $\mathbf{2}^{[n]}$ may be partitioned into $\binom{n}{\lfloor n/2 \rfloor}$ chains of size at least $d(n)$. (For comparison, a positive answer to Füredi's question would imply that the same result holds for some $d(n) \sim \sqrt{\pi/2}\sqrt{n}$.) More precisely, we first show that for $0 \leq j \leq n$, the union of the lowest $j + 1$ elements from each of the chains in the CSCD of $\mathbf{2}^{[n]}$ forms a poset $\mathbf{T}_j(n)$ with the normalized matching property and log-concave rank numbers. We then use our results on $\mathbf{T}_j(n)$ to show that the nodes in the CSCD chains of size less than $2d(n)$ may be repartitioned into chains of large minimum size, as desired.

EXPOSITORY ARTICLES

[17] *Rational nonaxis points on the unit circle have irrational angles*, Amer. Math. Monthly **123** (2016), no. 5, 490.

We give a short elementary proof that any rational point on the unit circle that does not lie on a coordinate axis has an angle that is an irrational multiple of π.

[18] (with J. H. Conway) *Some very interesting sequences*, in T. Shubin, D. F. Hayes, and G. Alexanderson (eds.), *Expeditions in Mathematics*, MAA Spectrum series, chapter 6, 75–86. MAA, Washington, DC, 2011.

In this expository paper, aimed at talented high school students and mathematical enthusiasts of all ages, we discuss a few of our favorite sequences and their remarkable properties. Topics include the Fibonacci sequence, the FRACTRAN program for primes, the omnipresent Bernoulli numbers, and many others.

Moufang loops

[19] *Explicit constructions of code loops as centrally twisted products*, Math. Proc. Camb. Phil. Soc. **128** (2000), 223–232.

Code loops are certain Moufang loop extensions of doubly even binary codes that have many applications in finite group theory (e.g., Conway's construction of the Monster). We give several methods for explicitly constructing code loops as *centrally twisted products.* More specifically, after establishing some preliminary examples, we show how to use decompositions of codes, such as Turyn's construction of the Golay code or the decomposition of a cyclic code into minimal cyclic codes, to build code loops out of more familiar pieces, such as abelian groups, extraspecial groups, or the octonion loop.

[20] *Moufang loops of class 2 and cubic forms*, Math. Proc. Camb. Phil. Soc. **128** (2000), 197–222.

Let L be a Moufang loop that is centrally nilpotent of class 2. We first show that the nuclearly-derived subloop (normal associator subloop) L^* of L has exponent dividing 6. It follows that L_p (the subloop of L of elements of p-power order) is associative for $p > 3$. Next, a Moufang loop L is said to be a *small Frattini Moufang loop*, or SFML, if L has a central subgroup Z of order p such that L/Z is an elementary abelian p-group. (For instance, it follows from work of Chein and Goodaire that the SFM 2-loops are precisely the class of *code loops*, in the sense of Griess.) L/Z is thus given the structure of what we call a *coded vector space*, or CVS. (For $p > 2$, a CVS is a vector space with attached linear, bilinear, and trilinear forms, and for $p = 2$, a CVS is a vector space equipped with functions that act like the intersection structure of a doubly even binary code.) We show that every CVS may be obtained from an SFML in this way, and that two SFML's are isomorphic in a manner preserving the central subgroup Z if and only if their CVS's are isomorphic up to scalar multiple. Consequently, we also obtain a relatively explicit characterization of isotopy in SFM 3-loops, a characterization easily extended to Moufang loops of class 2 and exponent 3. Finally, we sketch a method for constructing any finite Moufang loop which is centrally nilpotent of class 2.

Coset representations of modular subgroups

[21] *Permutation techniques for coset representations of modular subgroups*, in L. Schneps (ed.), *Geometric Galois Actions II: Dessins d'Enfants, Mapping Class Groups and Moduli*, volume 243 of *LMS Lect. Notes*, 67–77. Cambridge Univ. Press, 1997.

This article surveys the use of coset representations (specifying a subgroup $G_1 < G$ by writing down the action of G on the cosets of G_1 in G) in working with dessins d'enfants and similar structures. Questions expressed in terms of subgroups may sometimes be solved more easily in terms of coset representations. For instance, the results of [22] are summarized and then extended to obtain algorithms for solving problems such as finding the congruence closure of a modular subgroup.

[22] *Identifying congruence subgroups of the modular group*, Proc. AMS **124** (1996), no. 5, 1351–1359.

A simple test for determining if a given subgroup of $PSL_2(Z)$ is a congruence subgroup is exhibited, and a detailed description of its implementation is given. A more "invariant" congruence test is also described.

Quilts

[23] *Quilts: Central extensions, braid actions, and finite groups*, volume 1731 of *Lect. Notes Math.*, Springer-Verlag, 2000.

Quilts (developed by Norton, Parker, Conway, and the author) are 2-complexes used to analyze actions and subgroups of the 3-string braid group and similar groups. This research monograph, which supersedes [26] and [27], establishes the fundamentals of quilts and discusses connections with central extensions, braid actions, and finite groups, especially in the context of the Monstrous Moonshine phenomenon. In particular, *Norton's action* of the 3-string braid group on pairs of elements (resp. triples of involutions) of a finite group is analyzed in detail; new results on central extensions of triangle groups, triangle group quotients, and covers of Seifert fibered 3-manifolds are discussed; several open questions from [26] are resolved; and some open problems are discussed.

[24] *Quilts, the 3-string braid group, and braid actions on finite groups: an introduction*, in J. Ferrar and K. Harada (eds.), *The Monster and Lie Algebras*, volume 7 of *Ohio State Univ. Math. Res. Inst. Pubs.*, 85–97. de Gruyter, 1998.

This article gives a brief introduction to the basic definitions and results on quilts (fully described in [23]). In particular, we show how quilts may be used to study Norton's action of B_3 on pairs of elements (resp. triples of involutions) of a finite group, which may be relevant to the Generalized Moonshine conjectures, and we work out some small examples.

[25] *Some quilts for the Mathieu groups*, in C. Dong and G. Mason (eds.), *Moonshine, the Monster, and Related Topics*, volume 193 of *Contemp. Math.*, 113–122. AMS, 1996.

Examples of quilts arising from the Mathieu groups are presented, and some associated presentations are examined. In particular, presentations for some interesting extensions of M_{11}, M_{12}, and $M_{21} = L_3(4)$ are exhibited.

[26] *Quilts, T-systems, and the combinatorics of Fuchsian groups*, PhD thesis, Princeton Univ., 1994.

[27] (with J. H. Conway) *Quilts and T-systems*, J. Alg. **174** (1995), 856–908.