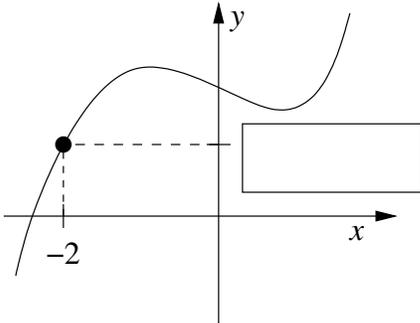


Sample Final Exam
Math 8, Spring 2019

1. (6 points) Find two functions f and g such that $(f \circ g)(x) = \sqrt{x^3 + x}$ and neither $f(x) = x$ nor $g(x) = x$. No explanation necessary.
2. (6 points) Find all **real** solutions to the equation $x^2 - 4x - 12 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).
3. (6 points) Solve the inequality $3 - 7x > 5$. You may express your answer either in interval notation (e.g., $[-16, 3] \cup (22, 55)$) or by inequalities (e.g., $x > 325$).
4. (6 points) Solve the following system of linear equations. Show all your work.

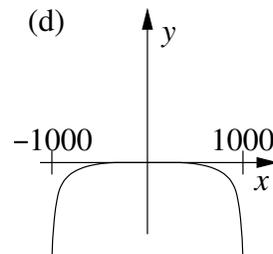
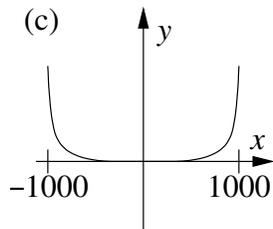
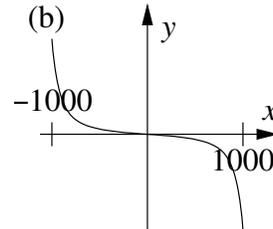
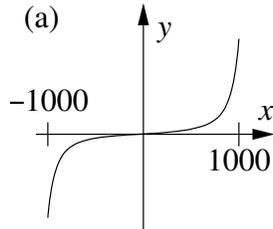
$$\begin{aligned}x - 5y - 2z &= -4 \\y + 3z &= 7 \\z &= -2\end{aligned}$$

5. (6 points) The graph below is a sketch of the graph of the function $f(x) = 2x^3 - 3x + 15$ (not to scale). Fill in the missing value on the y -axis. Show all your work.



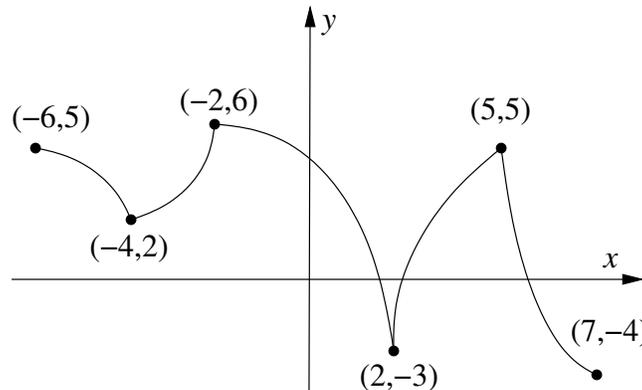
6. (6 points) Suppose f is a one-to-one function. If we know that $f(7) = -3$, what can we say about f^{-1} ? No explanation necessary.

7. (6 points) Consider the function $f(x) = 2x^6 - 77x^3$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly **EXPLAIN** how the **TWO** most important features of the coefficients and powers appearing in $f(x)$ justify your conclusion.



8. (6 points) Find all **real** solutions to the equation $2x^2 - 5x - 10 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).

9. (8 points) Suppose $f(x)$ is a function whose graph is shown below (not to scale).



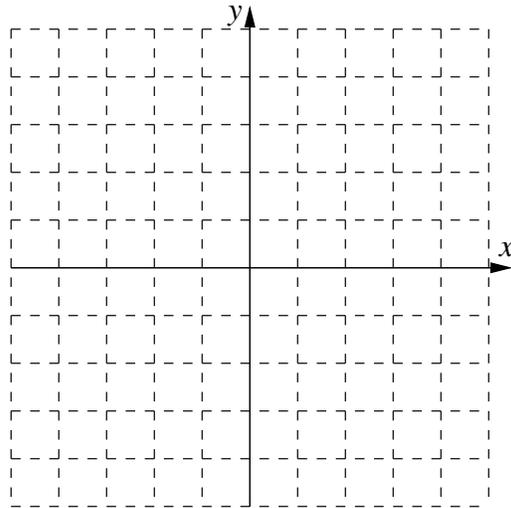
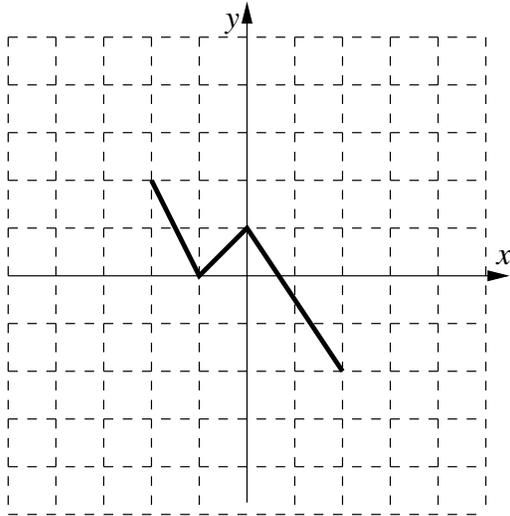
For the following, you may express interval answers either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”).

- On which interval or intervals (values of x) is the function $f(x)$ **decreasing**?
- At which **value(s)** of x does $f(x)$ have a **relative minimum**?

10. (8 points) Find the equation of the line through the points $(3, 7)$ and $(8, 5)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

11. (8 points) Draw the graph of $g(x) = 5^x$. Clearly label any x -intercepts, y -intercepts, or asymptotes.

12. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit \times 1 unit.) On the axes below right, graph the function $y = f(x + 3) - 1$, paying careful attention to the vertical and horizontal scales. No explanation necessary.



13. (8 points) Solve $\log_5(x - 7) = 2$ for x . Show all your work.

14. (8 points) Use long division to divide $f(x) = x^3 + 2x^2 - 3x - 5$ by $d(x) = x^2 - 2x + 3$. Show all your work, and express your final answer in the form $f(x) = q(x)d(x) + r(x)$.

15. (8 points) Find all solutions to the equation $\frac{1}{x^2 - 1} - \frac{5}{x - 1} = 2$. Show all your work, and leave your answer(s) in exact form (i.e., radicals and fractions, not decimals).

16. (8 points) Expand the expression $\log\left(\frac{(x - 5)^4(x^2 + 3)}{\sqrt{x + 7}}\right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.

17. (10 points) Write the quadratic function $f(x) = -2x^2 + 20x - 17$ in standard form and sketch its graph. Label the vertex and the y -intercept of your graph. (You do not need to label the x -intercept(s), if any.)

18. (10 points) Find the domain of the function $g(x) = \frac{\sqrt{x^2 - 4}}{x^2 - 2x - 3}$. Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

19. (10 points) Find all possible solutions to the following system:

$$\begin{aligned}3x + 5y &= 7, \\2x - y &= 5.\end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

20. (10 points) Let $f(x) = x^2 + 4x$. Simplify $\frac{f(7+h) - f(7)}{h}$ completely. Show all your work.

21. (12 points) Consider the polynomial function

$$f(x) = (x + 2)(x - 3)(x + 5)(x - 6)(x - 7)(x - 13).$$

- List the real zero(s) of f .
- Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

22. (12 points) Let

$$f(x) = x^3 - 5x^2 + 2x + 8.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

23. (12 points) J. K. Rowling has sold a total of 196,379 copies of her debut novel *Hairy Otter and the Storage Unit of Secrets*, when you combine sales from the hardcover and paperback editions. If each hardcover book sells for \$17 and each paperback sells for \$11, and the books have made a total revenue of \$2,650,525, how many hardcover books did she sell, and how many paperback books did she sell?

USE ALGEBRA, not guessing, to determine your answers. Show all your work and give your final answers in the form of a complete sentence, using the correct units, rounding off the numerical part of your answers to the nearest book (if necessary).

24. (12 points) Damian is saving up money for college, and currently has \$57,000 saved. He puts this money in an interest-bearing account whose balance t years from now will be

$$P(t) = 57000e^{0.06t}$$

dollars. How many years will it take Damian to reach his savings goal of \$120,000?

USE ALGEBRA, not guessing or calculator estimation, to determine the answer. Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.