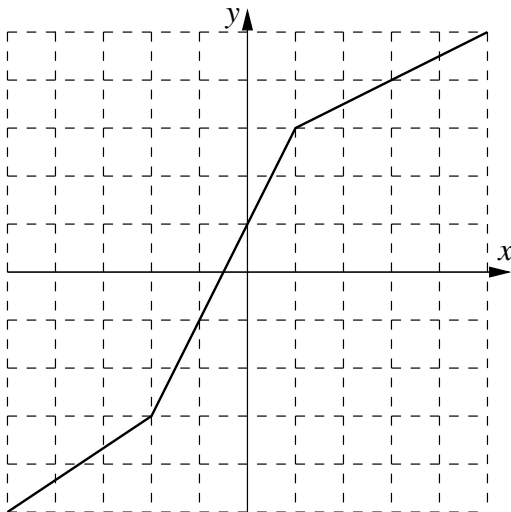


Sample Final Exam
Math 18A, Spring 2018

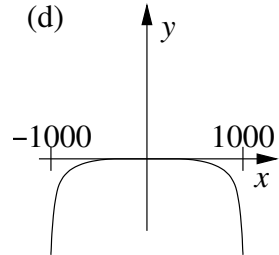
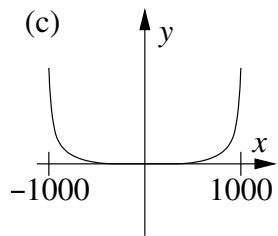
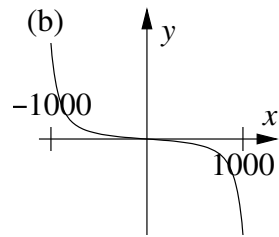
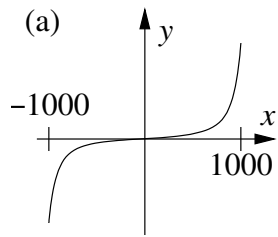
1. (6 points) Suppose f is a function whose graph is shown below. Find the value of $f^{-1}(2)$. No explanation necessary. You may assume that each square is 1 unit \times 1 unit, that all of the pieces of the graph that look like straight lines are actually straight lines, and that points on the graph that appear to be very close to grid points actually land on those points.



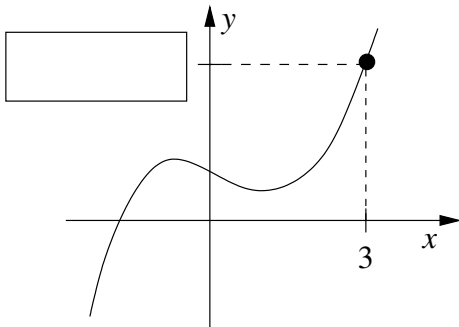
2. (6 points) Find all **real** solutions to the equation $2x^2 - 4x - 7 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).
3. (6 points) Find two functions f and g such that $(g \circ f)(x) = \sqrt[3]{x^2 + 5}$ and neither $g(x) = x$ nor $f(x) = x$. No explanation necessary.
4. (6 points) Solve the following system of linear equations. Show all your work.

$$\begin{aligned}x + 7y - z &= 11 \\y + 2z &= 7 \\z &= -4\end{aligned}$$

5. (6 points) Solve the inequality $3x - 4 > 7$. You may express your answer either in interval notation (e.g., $[-16, 3] \cup (22, 55)$) or by inequalities (e.g., $x > 325$).
6. (6 points) Find the x -intercept(s) of the graph of $xy^3 - 3x + y^2 = 17$. Show all your work, and leave your answer(s) in exact form.
7. (6 points) Consider the function $f(x) = 2x^5 - 157x^2$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.



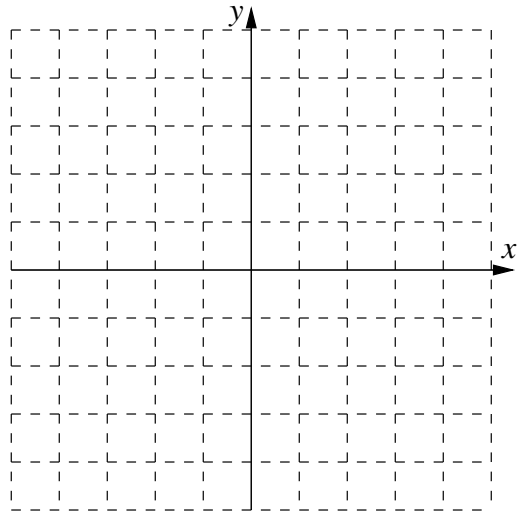
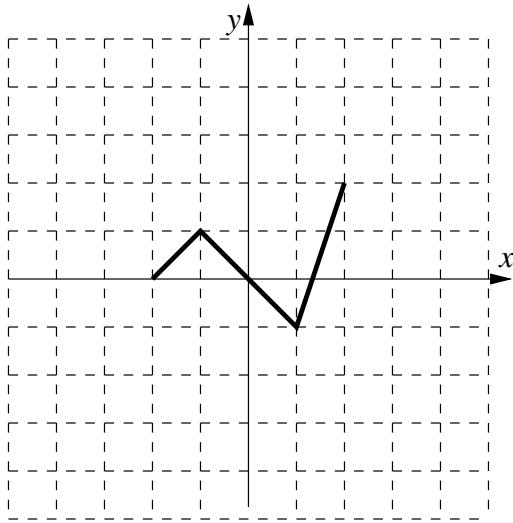
8. (6 points) The graph below is a sketch of the graph of the function $f(x) = x^3 - 2x + 3$ (not to scale). Fill in the missing coordinate of the indicated point. Show all your work.



9. (8 points) Use long division to divide $f(x) = 2x^3 - 5x^2 + 7x + 1$ by $d(x) = x^2 - x + 2$. Show all your work, and express your final answer in the form $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

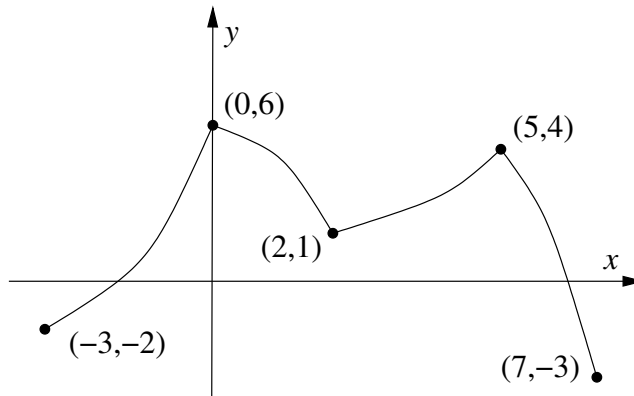
10. (8 points) Find the equation of the line through the points $(-3, 7)$ and $(4, 2)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

11. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit \times 1 unit.) On the axes below right, graph the function $y = f(x + 2) + 3$, paying careful attention to the vertical and horizontal scales. No explanation necessary.



12. (8 points) Draw the graph of $g(x) = 1.75^x$. Clearly label any x -intercepts, y -intercepts, or asymptotes.

13. (8 points) Suppose $f(x)$ is a function whose graph is shown below (not to scale).



For the following, you may express interval answers either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”).

(a) On which interval or intervals (values of x) is the function $f(x)$ **decreasing**?

Interval(s) where $f(x)$ is decreasing:

(b) At which **value(s) of x** does $f(x)$ have a **relative minimum**?

Value(s) of x where $f(x)$ has a relative minimum:

14. (8 points) Solve $\log_2(x - 5) = 7$ for x . Show all your work.

15. (8 points) Expand the expression $\log\left(\frac{(x-3)^7(x+2)^5}{\sqrt{x^2+11}}\right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.

16. (8 points) Find all solutions to the equation $\frac{5}{x-7} + 3 = x$. Show all your work, and leave your answer(s) in exact form (i.e., radicals and fractions, not decimals).

17. (10 points) Let $f(x) = 3x^2 - 4$. Simplify $\frac{f(2+h) - f(2)}{h}$ completely. Show all your work.

18. (10 points) Write the quadratic function $f(x) = x^2 + 6x - 13$ in standard form and sketch its graph. Label the vertex and the y -intercept of your graph. (You do not need to label the x -intercept(s), if any.)

19. (10 points) Find the domain of the function $g(x) = \frac{x-13}{\sqrt{x^2-x-6}}$. Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

20. (10 points) Find all possible solutions to the following system:

$$3x + 5y = 4,$$

$$2x - 7y = 7.$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

21. (12 points) Consider the polynomial function $f(x) = (x-2)(x-5)(x+7)(x+9)$.

(a) List the real zero(s) of f .

(b) Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

22. (12 points) The latest hip-hop musical about America’s founders, *Gouverneur!: the Gouverneur Morris Experience*, has just released its latest song for streaming, “(I’m Not Throwing Away) My Pen”. A mathematical model predicts that t days after its release, the song will have been played

$$P(t) = 23000e^{0.04t}$$

times on streaming music services. In other words, $P(t)$ is the number of times the song will have been played t days after its release.

How many days after the song is released will it have been played 1,000,000 (one million) times? Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

23. (12 points) Let

$$f(x) = x^3 - 6x^2 + 5x + 12.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

24. (12 points) The movie *JustUs League* made a lot of money, but not as much as its sequel, *JustUs League 2: Now With More of That Wonder Lady*. In fact, the second movie (*JustUs League 2*) made 15% more money than the original movie (*JustUs League*).

If the two movies together made a total of \$743.8 million dollars, how much money did the original movie (*JustUs League*) make? **USE ALGEBRA**, not guessing, to determine the answer. Show all your work and give your final answer in the form of a complete sentence, using the correct units, rounding off the numerical part of your answer to the nearest .01 million dollars.