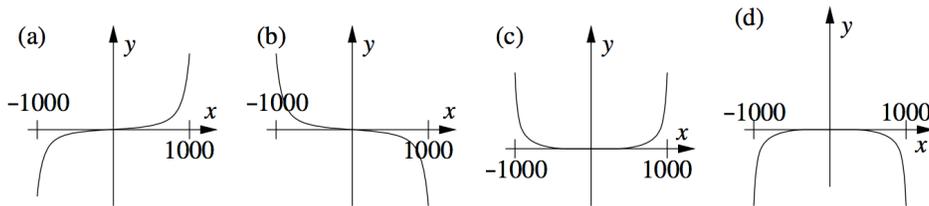


Sample Final Exam
Math 8, Fall 2017

This test consists of 24 questions on 3 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (6 points) Find the x -intercept(s) of the equation $2x^2 + 2xy + 3y^2 = 8$. Show all your work, and leave your answer in exact form.
2. (6 points) Solve the inequality $6 < 4x - 2 < 10$. You may express your answer either in interval notation (e.g., " $[-16, 3] \cup (22, 55)$ ") or by inequalities (e.g., " $x > 325$ ").
3. (6 points) Let $f(x) = 5 - 2x$. Find a formula for the inverse function f^{-1} . Show all your work.
4. (6 points) Simplify $\log_5 \frac{1}{\sqrt{5}}$. No explanation necessary.
5. (6 points) Consider the function $f(x) = -2x^6 + 22x^2$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.



6. (6 points) Solve the following system of linear equations. Show all your work.

$$\begin{aligned}x - 2y + 3z &= 7 \\2x + y + z &= 4 \\z &= 1\end{aligned}$$

7. (6 points) Find all **real** solutions to the equation $2x^2 - 4x - 3 = 0$. If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form.

8. (6 points) Find two functions f and g such that $(g \circ f)(x) = \frac{1}{\frac{x}{3-x} + 1}$ and neither $g(x) = x$ nor $f(x) = x$. No explanation necessary.

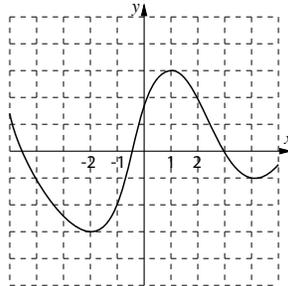
9. (6 points) Is the point $(2, -4)$ on the graph of the function $f(x) = 2x - 3x^2$? Answer YES or NO clearly, briefly (one sentence) **JUSTIFY** your answer, and show all your work.

10. (8 points) Expand the expression $\log \left(\frac{x^3 \sqrt{x+3}}{(x+1)^2(x-1)} \right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.

11. (8 points) Use long division to divide $f(x) = 4x^3 - 13x^2 + 2x - 7$ by $d(x) = x^2 + 3x - 2$. Show all your work, and express your final answer in the form $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$.

12. (8 points) Find the equation of the line through the points $(-13, -5)$ and $(1, 2)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

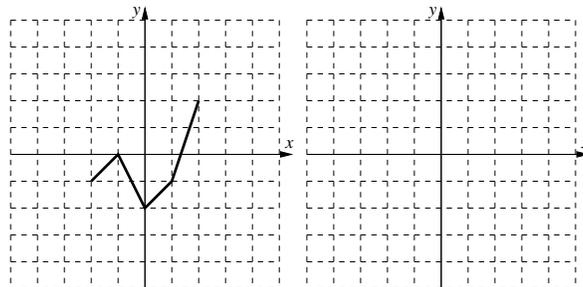
13. (8 points) Suppose $f(x)$ is a function whose graph is shown below.



For the following, you may express interval answers either in interval notation (e.g., “on the intervals $(22, 55)$ and $[-16, 3]$ ”) or by inequalities (e.g., “for $x > 325$ ”).

- (a) On which interval or intervals (values of x) is the function $f(x)$ **increasing**?
- (b) At which **value(s)** of x does $f(x)$ have a **relative minimum**?

14. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit \times 1 unit.) On the axes shown below right, graph the function $y = f(x - 1) + 2$ paying careful attention to the vertical and horizontal scales.



15. (8 points) Draw the graph of $g(x) = 2\left(\frac{1}{3}\right)^x$. Clearly label any x -intercepts, y -intercepts, or asymptotes.

16. (10 points) Solve the following inequality:

$$(x - 1)(x + 50)(x + 51) > 0.$$

Show all your work.

17. (10 points) Find the domain of the function $f(x) = \sqrt{x^2 - 5x + 4}$. Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

18. (10 points) Let $f(x) = 3x^2 - 2x$. Simplify $\frac{f(1+h) - f(1)}{h}$ completely. Show all your work.

19. (10 points) Write the quadratic function $f(x) = -x^2 + 6x - 4$ in standard form and sketch its graph. Label the vertex and the y -intercept of your graph. (You do not need to label the x -intercept(s), if any.)

20. (10 points) Find all possible solutions to the following system:

$$\begin{aligned}2x - 3y &= 8, \\ -x + 2y &= -4.\end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

21. (12 points) You invest \$200,000 in the bank, in an account that pays 5% interest, compounded continuously. The balance in your account t years after you open it will be

$$P(t) = 200000e^{0.05t}$$

How many years after you open the account will it take for the account balance to reach \$300,000?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

22. (12 points) Let

$$f(x) = x^3 + 3x^3 - 6x - 18.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

23. (12 points) Consider the polynomial function $f(x) = (x+2)(x-1)(x-4)(x+5)(x-3)$.

- List the real zero(s) of f .
- Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

24. (12 points) Mr. and Mrs. Packed-Man had a record breaking month, and ate a total of 642,831 pellets. Mrs. Packed-Man ate 10% more than Mr. Packed-Man did. How many pellets did Mr. Packed-Man eat?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.