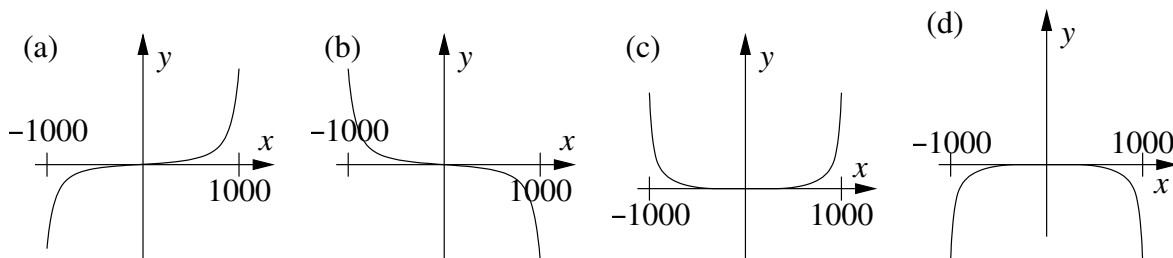


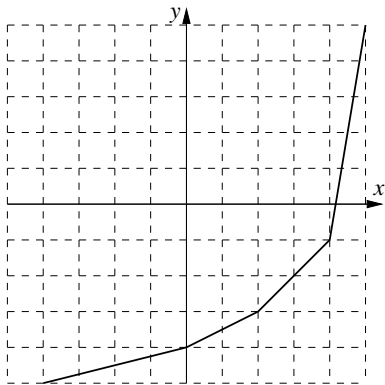
**Sample Final Exam**  
**Math 8, Fall 2016**

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE**  $3 \times 5$  notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

- (6 points) Find the  $x$ -intercept(s) of the graph of  $y^2 - 3x + 4y = 5$ . Show all your work, and leave your answer(s) in exact form.
- (6 points) Consider the function  $f(x) = 3x^4 - 25x^3$ . Which of the graphs below best matches the graph of  $f(x)$ ? Note that the horizontal scale on the graph goes from  $x = -1000$  to  $x = 1000$ , and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.



- (6 points) Suppose  $f$  is a function whose graph is shown below. Find the value of  $f^{-1}(2)$ . No explanation necessary. You may assume that each square is  $1 \text{ unit} \times 1 \text{ unit}$ , that all of the pieces of the graph that look like straight lines are actually straight lines, and that points on the graph that appear to be very close to grid points actually land on those points.



- (6 points) Simplify  $\log_7(\sqrt[3]{7})$ . No explanation necessary.
- (6 points) Solve the following system of linear equations. Show all your work.

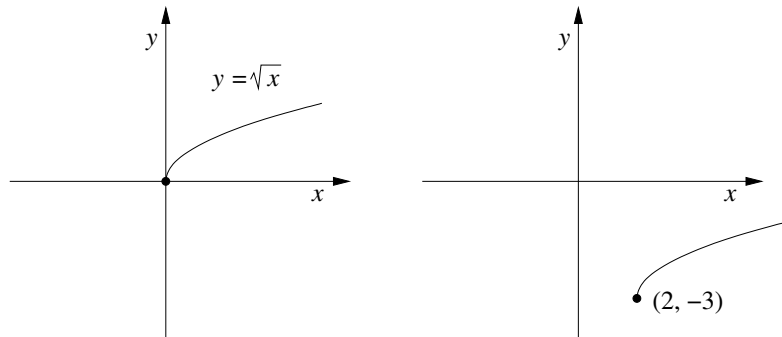
$$\begin{aligned} x - 3y + 5z &= -1 \\ y + 2z &= 7 \\ z &= 3 \end{aligned}$$

6. (6 points) Find all **real** solutions to the equation  $x^2 - 5x - 13 = 0$ . If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form.

7. (6 points) Find two functions  $f$  and  $g$  such that  $(g \circ f)(x) = \sqrt{1 - x^5}$  and neither  $g(x) = x$  nor  $f(x) = x$ . No explanation necessary.

8. (6 points) Solve the linear inequality  $3x - 5 \leq 17$ . You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

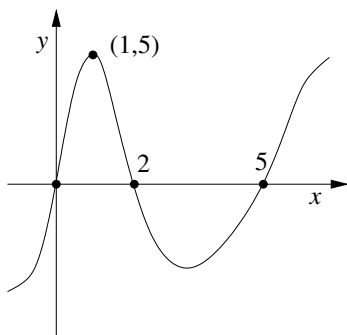
9. (8 points) Consider the two graphs below. The left-hand graph is the graph of  $y = \sqrt{x}$ , and the right-hand graph is the graph of  $y = \sqrt{x}$ , shifted 2 units to the right and 3 units down. Use transformations to write an equation for the right-hand graph. No explanation necessary.



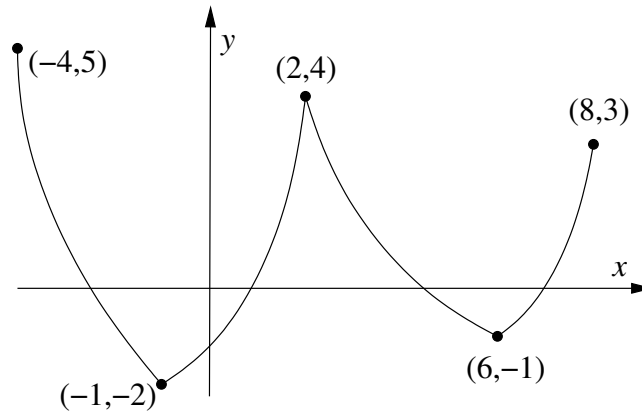
10. (8 points) Find the equation of the line through the points  $(-2, 7)$  and  $(3, 11)$ . Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

11. (8 points) Find all solutions to the equation  $\frac{1}{x+1} - \frac{10}{x-3} = 1$ . Show all your work, and leave your answer(s) in exact form.

12. (8 points) The graph of a polynomial function  $y = f(x)$  is shown below (not to scale). Suppose also that  $f(x) = (x - 5)q(x) + r$ , where  $q(x)$  is a polynomial and  $r$  is a real number. What does the Remainder Theorem tell you about the value of  $r$ ? **EXPLAIN** your answer in **ONE SENTENCE**, indicating the **ONE** aspect of the graph of  $y = f(x)$  that justifies your answer.



13. (8 points) Suppose  $f(x)$  is a function whose graph is shown below (not to scale).



For the following, you may express interval answers either in interval notation (e.g., “on the intervals  $(22, 55)$  and  $[-16, 3]$ ”) or by inequalities (e.g., “for  $x > 325$ ”).

- (a) On which interval or intervals (values of  $x$ ) is the function  $f(x)$  **decreasing**?
- (b) At which **value(s) of  $x$**  does  $f(x)$  have a **relative maximum**?
14. (8 points) Expand the expression  $\log\left(\frac{(x^2 + 1)(x - 7)^{13}}{(x - 5)^3}\right)$  as a sum, difference, and/or multiple of logarithms. Show all your work.
15. (8 points) Draw the graph of  $g(x) = \left(\frac{3}{2}\right)^x$ . Clearly label any  $x$ -intercepts,  $y$ -intercepts, or asymptotes.
16. (8 points) Use long division to divide  $f(x) = 7x^3 - 3x^2 + 5x + 13$  by  $d(x) = x^2 + x - 1$ . Show all your work, and express your final answer in the form  $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ .
17. (10 points) Let  $f(x) = 5x^2 + x$ . Simplify  $\frac{f(7+h) - f(7)}{h}$  completely. Show all your work.
18. (10 points) Find all possible solutions to the following system:

$$\begin{aligned} 11x - 3y &= 2, \\ 2x - y &= 7. \end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

19. (10 points) Write the quadratic function  $f(x) = x^2 + 10x - 7$  in standard form and sketch its graph. Label the vertex and the  $y$ -intercept of your graph. (You do not need to label the  $x$ -intercept(s), if any.)

**20.** (10 points) Find the domain of the function  $g(x) = \frac{1}{\sqrt{x^2 - 3x - 4}}$ . Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

**21.** (12 points) In calendar year 2015, retired rock stars Kim and Francis together made a total of \$285,571.60. Also, Kim made 15% more than Francis did in 2015. How much money did Francis make in 2015?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

**22.** (12 points) The evil Lord Bolton invests \$150,000 in the Totally Secret National Bank of Panama, in an account that pays 7% interest, compounded continuously. The balance in his account,  $t$  years after he opens it, will be

$$P(t) = 150000e^{0.07t}.$$

How many years after Lord Bolton opens the account will it take for the account balance to reach \$270,000?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

**23.** (12 points) Let

$$f(x) = x^3 - 19x + 30.$$

Find the rational zeros of  $f(x)$  by factoring  $f(x)$  completely. Show all your work. Make sure you include both the complete list of zeros of  $f(x)$  and the factorization of  $f(x)$  in your final answer.

**24.** (12 points) Consider the polynomial function  $f(x) = x(x + 1)(x - 2)(x + 7)$ .

- (a) List the real zero(s) of  $f$ .
- (b) Sketch the graph of  $f(x)$ . In particular, make sure that the above information about zeros is clearly visible in your graph.