**Sample Final Exam**
Math 8, Spring 2016

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (6 points) Find the y-intercept(s) of the graph of $y^2 - x^3 + 1 = 7$. Show all your work, and leave your answer(s) in exact form.

2. (6 points) Solve the linear inequality $3 - 2x > 18$. You may express your answer either in interval notation (e.g., "$[-16, 3] \cup (22, 55)$") or by inequalities (e.g., "$x > 325$").

3. (6 points) Simplify $\log_5 \left(\dfrac{1}{5^8} \right)$. No explanation necessary.

4. (6 points) Let $f(x) = x^2$ and $g(x) = 2x - 7$. Calculate $(g \circ f)(-3)$. Show all your work.

5. (6 points) Solve the following system of linear equations. Show all your work.

$$x - y + 4z = 13$$
$$y + 3z = -3$$
$$z = 2$$

6. (6 points) Find all solutions to the equation $x^2 - 6x + 4 = 0$. Show all your work, and leave your answer(s) in exact form.

7. (6 points) Consider the function $f(x) = 7x^5 - 100x^2 + 13$. Which of the graphs below best matches the graph of $f(x)$? Note that the horizontal scale on the graph goes from $x = -1000$ to $x = 1000$, and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.

(a)

(b)

(c)

(d)
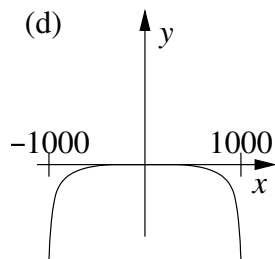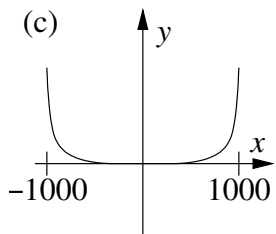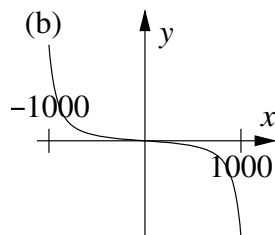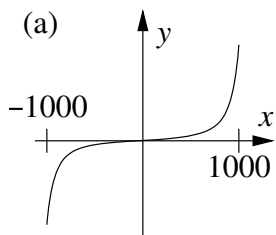
8. (6 points) Find two functions f and g such that $(f \circ g)(x) = \dfrac{1}{x^2 - 3}$ and neither $f(x) = x$ nor $g(x) = x$. No explanation necessary.

9. (8 points) Draw the graph of $g(x) = 2(3^x)$. Clearly label any x-intercepts, y-intercepts, or asymptotes.

10. (8 points) Use long division to divide $2x^3 - 4x^2 + x + 7$ by $x^2 + x - 2$. Show all your work, and express your final answer in the form $\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$.

11. (8 points) Suppose $f(x)$ is a function whose graph is shown below (not to scale).
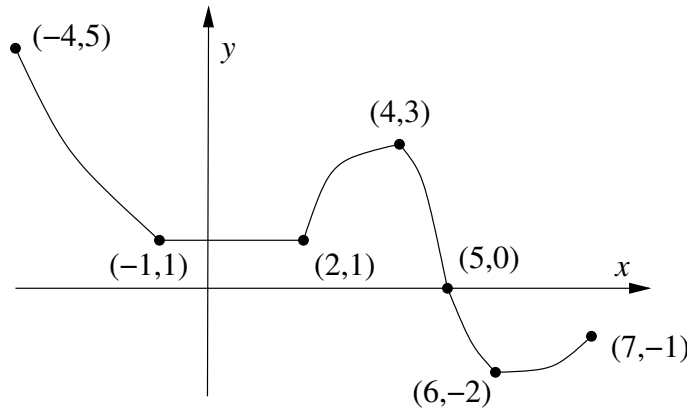
For the following, you may express your answer either in interval notation (e.g., "on the intervals $(22, 55)$ and $[-16, 3]$") or by inequalities (e.g., "for $x > 325$").

(a) On which interval or intervals (values of x) is the function $f(x)$ **decreasing**?

(b) On which interval or intervals (values of x) is the function $f(x)$ **constant**?

12. (8 points) Expand the expression $\log\left(\dfrac{x^5(x-3)^2}{\sqrt{x+2}}\right)$ as a sum, difference, and/or multiple of logarithms. Show all your work.

13. (8 points) Solve the equation $\dfrac{6}{x-5} + 7 = \dfrac{2x}{x-5}$. Show all your work, and leave your answer in exact (fractional) form.

14. (8 points) Let $y = f(x)$ be the function whose graph is shown below left. (Each square is 1 unit $\times$ 1 unit.) On the axes below right, graph the function $y = f(x + 1) - 3$, paying careful attention to the vertical and horizontal scales. No explanation necessary.
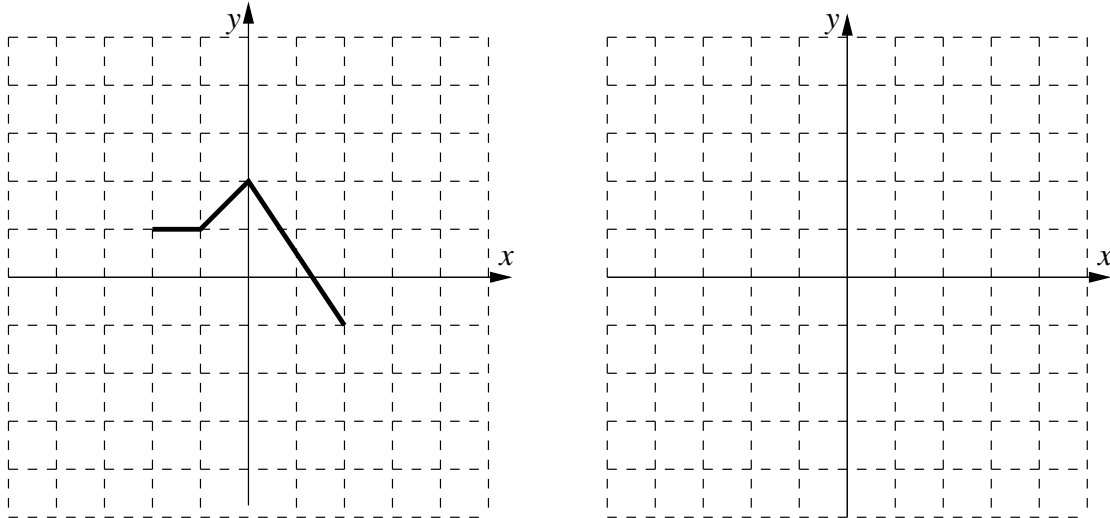
15. (8 points) Let $f(x) = 7x - 5$. Find a formula for the inverse function f^{-1}. show all your work.

16. (8 points) Find the equation of the line through the points $(5, 3)$ and $(-7, 16)$. Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

17. (10 points) Let $f(x) = x^2 + x$. Simplify $\dfrac{f(3 + h) - f(3)}{h}$ completely. Show all your work.

18. (10 points) Write the quadratic function $f(x) = x^2 - 6x - 5$ in standard form and sketch its graph. Label the vertex and the y-intercept of your graph. (You do not need to label the x-intercept(s), if any.)

19. (10 points) Find all possible solutions to the following system:

$$5x - 7y = 3,$$
$$-x + 2y = 1.$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

20. (10 points) Find the domain of the function $g(x) = \sqrt{x^2 - 7x - 18}$. Show all your work. You may express your answer either in interval notation (e.g., "$[-16, 3] \cup (22, 55)$") or by inequalities (e.g., "$x > 325$").

21. (12 points) When the ice planet Hooth is first settled ($t = 0$), its population is 300,000. After that, the population $P(t)$ of Hooth t years after that first settlement is modelled by

$$P(t) = 300000e^{.02t}.$$

How many years does it take after the initial settlement for the population of Hooth to reach 400,000?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

22. (12 points) Consider the polynomial function $f(x) = (x-1)(x+2)(x-3)(x-4)(x+5)$.

(a) List the real zero(s) of f.

(b) Sketch the graph of $f(x)$. In particular, make sure that the above information about zeros is clearly visible in your graph.

23. (12 points) Let
$$f(x) = x^3 + 4x^2 + x - 6.$$

Find the rational zeros of $f(x)$ by factoring $f(x)$ completely. Show all your work. Make sure you include both the complete list of zeros of $f(x)$ and the factorization of $f(x)$ in your final answer.

24. (12 points) The movie studio Phony Pictures releases the film *The Mango Express* online. Viewers can purchase the movie for $15 and rent the movie for $6. The studio counts a total of 2 million (2,000,000) transactions (purchases plus rentals) over the first week of release, and collects a total of $15 million ($15,000,000) of revenue in that first week.

How many rentals were there in the first week of release? Show all your work, and give your final answer in the form of a complete sentence, using the correct units. You may either round off your final numerical answer to the nearest whole number or leave it in exact (fractional) form.