

**Sample Final Exam**  
**Math 8, Fall 2015**

This test consists of 24 questions on 4 pages (including this cover sheet), totalling 200 points. You are allowed to use the usual calculators and **ONE**  $3 \times 5$  notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

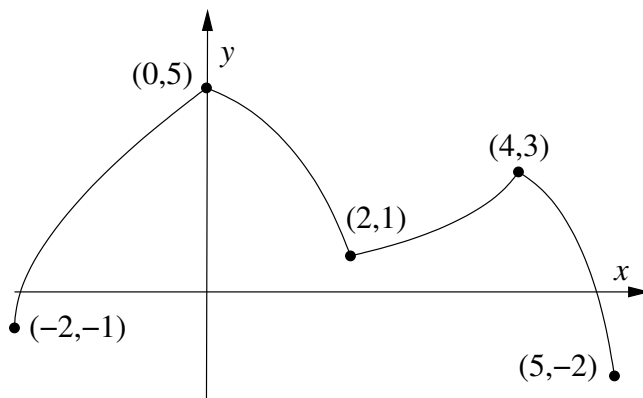
- (6 points) Let  $f(x) = x^2 - 7$  and  $g(x) = 2x + 5$ . Find the value of  $f(g(-3))$ . Show all your work.
- (6 points) Find the  $x$ -intercept(s) of the graph of  $x + x^2y^2 - 7y + 5 = 0$ . Show all your work.
- (6 points) Solve the equation  $\frac{2}{x-3} = \frac{7}{x+5}$ . Show all your work, and leave your answer in exact (fractional) form.
- (6 points) Simplify  $\log_7(7^{15}7^{32})$ . Show all your work.
- (6 points) Find two functions  $f$  and  $g$  such that  $(f \circ g)(x) = \sqrt{1 + e^{3x}}$  and neither  $f(x) = x$  nor  $g(x) = x$ . No explanation necessary.
- (6 points) Solve the following system of linear equations. Show all your work.

$$x + 5y + 3z = 7$$

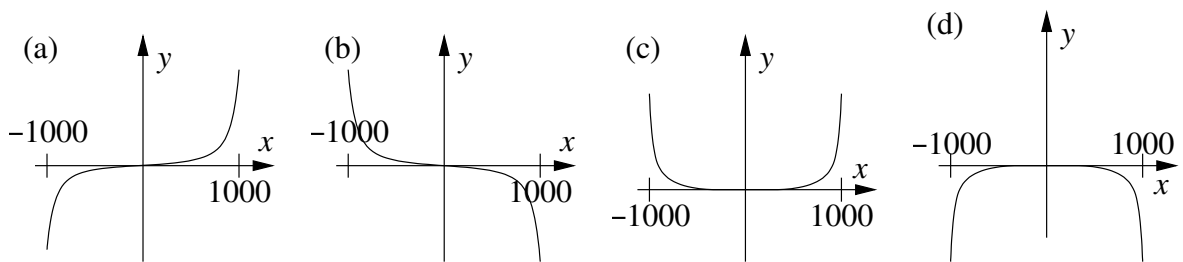
$$y - 2z = 4$$

$$z = 1$$

- (6 points) Suppose  $f(x)$  is a function whose graph is shown below (not to scale). On which interval or intervals (values of  $x$ ) is the function  $f(x)$  **increasing**? You may express your answer either in interval notation (e.g., “on the intervals  $(22, 55)$  and  $[-16, 3]$ ”) or by inequalities (e.g., “for  $x > 325$ ”).



- (6 points) Consider the function  $f(x) = 2x^4 - x^3 + 12x + 35$ . Which of the graphs below best matches the graph of  $f(x)$ ? Note that the horizontal scale on the graph goes from  $x = -1000$  to  $x = 1000$ , and the vertical scale is unspecified. Circle your answer, and briefly (1–3 sentences) **EXPLAIN** why the graph you chose is the best match.



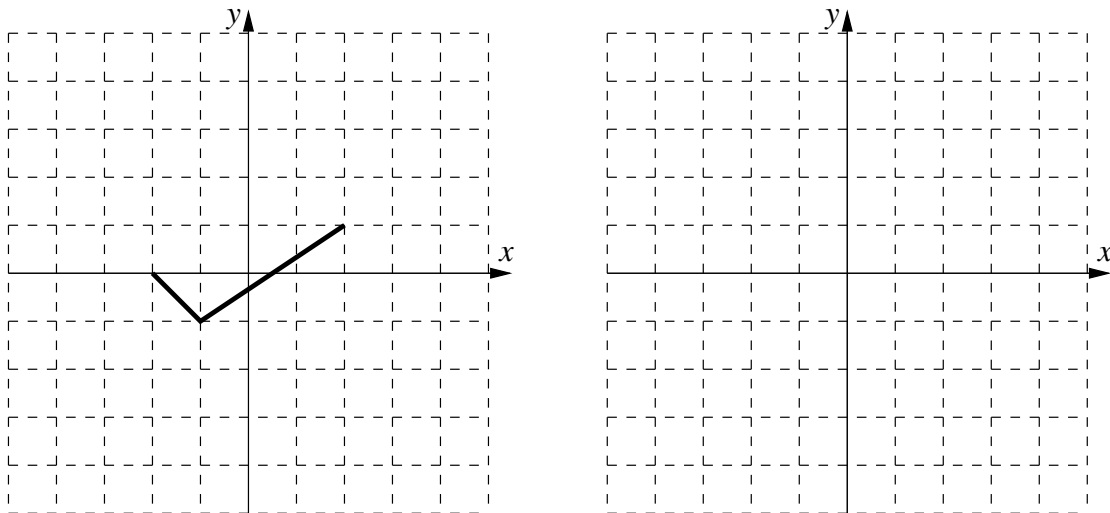
9. (8 points) Use long division to divide  $x^3 - 3x^2 + 7x + 5$  by  $x^2 - x + 2$ . Show all your work, and express your final answer in the form  $\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$ .

10. (8 points) Find the equation of the line through the points  $(3, -7)$  and  $(12, 6)$ . Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

11. (8 points) Solve the linear inequality  $2x + 5 \leq 7x - 8$ . You may express your answer either in interval notation (e.g.,  $[-16, 3] \cup (22, 55)$ ) or by inequalities (e.g.,  $x > 325$ ).

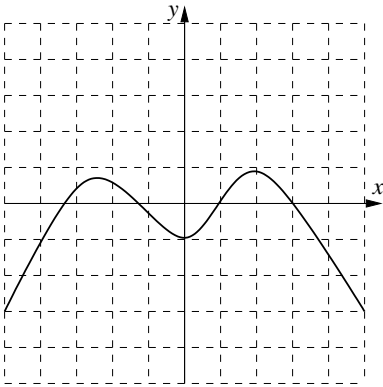
12. (8 points) Solve the equation  $x = \sqrt{5x - 6}$ . Show all your work.

13. (8 points) Let  $y = f(x)$  be the function whose graph is shown below left. (Each square is 1 unit  $\times$  1 unit.) On the axes below right, graph the function  $y = 2 + f(x - 3)$ , paying careful attention to the vertical and horizontal scales. No explanation necessary.

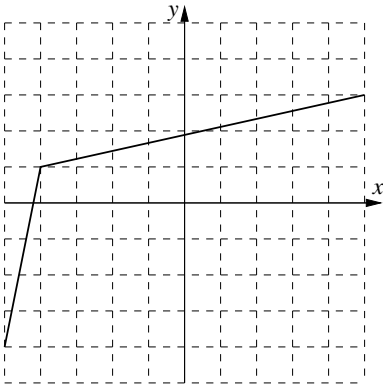


14. (8 points) Condense the expression  $4 \log(x + 5) + 2 \log x - 3 \log(x - 2)$  to the logarithm of a single quantity. Show all your work.

15. (8 points) Suppose  $f(x)$  is a polynomial whose graph is (partially) shown below. (Each square is 1 unit  $\times$  1 unit.) If we divide  $f(x)$  by  $(x - 3)$ , what is the remainder? Briefly (one sentence) **EXPLAIN** your answer, possibly by circling or otherwise indicating a portion or portions of the graph that justify your answer.



16. (8 points) Suppose  $f$  is a function whose graph is shown below. Draw the graph of the inverse function  $f^{-1}$  on the same axes. No explanation necessary.



17. (10 points) Let  $g(x) = 2^{-x}$ .

- Draw the graph of  $y = g(x) = 2^{-x}$ . Clearly label any  $x$ -intercepts,  $y$ -intercepts, or asymptotes.
- Find the domain and the range of  $g(x)$ . You may express your answers either in interval notation (e.g., “on the intervals  $(22, 55)$  and  $[-16, 3]$ ”) or by inequalities (e.g., “ $x > 325$ ” or “ $-17 \leq y \leq 3$ ”). No explanation necessary.

18. (10 points) Find the domain of the function  $g(x) = \frac{\sqrt{x+7}}{x^2-9x}$ . Show all your work.

You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

19. (10 points) Let  $f(x) = 3x^2$ . Simplify  $\frac{f(5+h) - f(5)}{h}$  completely. Show all your work.

20. (10 points) Write the quadratic function  $f(x) = x^2 - 8x + 9$  in standard form and sketch its graph. Label the vertex and the  $y$ -intercept of your graph. (You do not need to label the  $x$ -intercept(s), if any.)

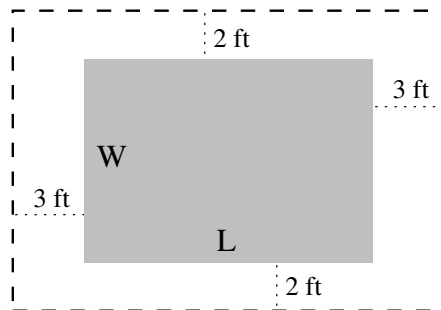
21. (12 points) Let

$$f(x) = x^3 - 5x^2 + 2x + 8.$$

Find the rational zeros of  $f(x)$  by factoring  $f(x)$  completely. Show all your work. Make sure you include both the complete list of zeros of  $f(x)$  and the factorization of  $f(x)$  in your final answer.

**22.** (12 points) Edie and Micah are building a rectangular swimming pool completely surrounded by a rectangular fence, as shown below. Suppose that:

- The pool is longer than it is wide (i.e.,  $L > W$ ).
- There is a 3 foot gap between the fence and the shorter sides of the pool and a 2 foot gap between the fence and the longer sides of the pool.
- The area of the pool (shaded area) is 150 square feet.
- The length of the fence (i.e., the perimeter of the larger, outside rectangle, indicated by the heavy dashed lines in the picture) is 80 feet.



Find the dimensions  $L$  and  $W$  of the pool. Show all your work, round off your final numerical answers to the nearest .01, if necessary, and give your final answers in the form of a complete sentence or sentences, using the correct units. Note that the correct numerical answers may not be integers (whole numbers).

**23.** (12 points) Consider the polynomial function  $f(x) = (x^2 - 1)(x^2 - 9)$ .

- Factor  $f(x)$  completely.
- List the real zero(s) of  $f$ .
- Sketch the graph of  $f(x)$ . In particular make sure that the above information about zeros is clearly visible in your graph.

**24.** (12 points) The Internet top-level domain name `.troll` is created to host annoying web pages. The `.troll` domain starts with 5000 active web pages, and the number of active web pages on `.troll` after  $t$  days of its existence is

$$W(t) = 5000e^{.01t}.$$

(Just in case your exam copy is unclear: The exponent in the above equation is  $.01t = 0.01t$ .)

If the `.troll` domain can only contain a maximum of 10,000,000 web pages, after how many days of its existence will `.troll` run out of space for web pages?

Show all your work, round off your final numerical answer to the nearest .01, if necessary, and give your final answer in the form of a complete sentence, using the correct units.