Topics to be covered in Math 18A final
Fall 2020

General information. Because all Math 18A students will be taking a common final this semester, here is a short summary of the semester. Everything on this list is fair game for the final.

Also, to make things clear in advance, here are the ground rules under which the final exam will be administered:

- The final exam will be a stricly timed test of 2 hours, 15 minutes. It will be held on **Sat Dec 12** from **11:30am–1:45pm**.
- Students will be allowed to use a calculator for the exam, but **not** a graphing calculator (TI-83, 84, 85, 86) and **not** a calculator that can do algebra (a "computer algebra system"), like the TI-89 or TI-92.
- Each student will be allowed to write notes, of any kind, on **ONE** 3×5 note card (both sides).
- Each student **MUST BRING THEIR STUDENT ID CARD.** Students without ID will receive 0's.
- Students must turn in their ID cards to their instructor to receive a final exam, and must turn in their exam to get their ID cards back.
- Each instructor will take attendance for their section and inspect calculators and note cards for compliance.
- Students will not be required to simplify their answers unless given specific instructions; in fact, for certain questions, students may be asked **not** to simplify their answers. Students may also be asked to leave answers in exact form (e.g., radicals, not decimal approximations).

As much as possible, exam problems will be written to be compatible with the problems in the text. However, instructors and students should not expect them to be exactly the same as text problems (with different numbers); nor should you expect the final to cover the same material as the sample final. The topics list below, however, should be a fairly reliable guide.

Chapter 0. No Chapter 0 material will be tested on its own on the final exam. Of course, students should be able to add and simplify fractions, understand negative and fractional exponents, and so on, in the service of material from Chapters 1–4. Topics that will be useful later that students typically have problems with include: negative and fractional exponents, laws of exponents, $(a \pm b)^2$, manipulating and simplifying rational functions (fractions of polynomials), factoring.

Section 1.1. Linear equations and how to solve them. Solving fractional equations by reducing them to linear equations.

Section 1.2. "Mathematical modeling" (i.e., word problems). Students will not be tested on word problems with well-known canned solutions (e.g., mixture problems); students will instead be asked to solve relatively straightforward problems where they nevertheless need to read and understand the problem to solve it.

Section 1.3. Quadratic equations; zero-factor property. Solving quadratic equations by factoring and taking square roots.

Section 1.4. Using the quadratic formula. Discriminant and number of solutions.

Section 1.5. Solving other equations: Higher-degree polynomials by factoring; quadratic-ish equations; equations with radicals; fractional equations; absolute value equations.

Section 1.6. Solving linear inequalities. Absolute value inequalities via $|y| < a$ if and only if $-a < y < a$, etc.

Section 1.7. Solving nonlinear inequalities by factoring and test intervals. Rational inequalities; domain of a radical expression.

Section 2.1. Points in the plane. Distance and midpoint formulas. Graph of an equation. Definition of x-intercepts and y-intercepts; finding x- and y-intercepts. Symmetries of graphs: with respect to x-axis, y-axis, origin. Equation of a circle.

Section 2.2. Slope of a line. **Point-slope formula** for the equation of a line. (**Note:** On the final, students may leave equation of a line in point-slope form for full credit.) Slope-intercept form of equation of a line. Parallel and perpendicular lines and their slopes.

Section 2.3. NOT ON FINAL.

Section 2.4. Definition of function. Evaluating $f(2)$, $f(a)$, $f(a+h)$, etc. Evaluating piecewise defined functions (split formulas). Idea of domain and range of a function. Finding domain of a function given by a formula.

Section 2.5. The graph of a function: Reading function information from a graph; graphing given function information. Vertical line test. Definitions: increasing on an interval, decreasing on an interval, local maxima and minima. Recognizing increasing, decreasing, local max, local min on a graph. Graphs of even and odd functions.

Section 2.6. Idea of transformations of graphs. Vertical shifts; horizontal shifts; vertical and horizontal reflections/flips; vertical stretching and shrinking.

Section 2.7. Sums, differences, products, and quotients of functions and their domains. Compositions of functions and their domains: Basic examples, $f \circ g$ versus $g \circ f$. Computing $(f \circ g)(x)$: formulas, tables, graphs. Not on final: Finding domains of compositions. **Critically important for calculus:** Expressing $H(x)$ as $(f \circ g)(x)$. **This will be on the Fall 2020 final.**

Section 3.1. Graphic quadratics by transformations of $y = x^2$. Standard form of a quadratic function: graphing. Finding standard form by completing the square. Finding minimum or maximum value of a quadratic. (You may use the fact that the max or min value always occurs at $x = -\dfrac{b}{2a}$, and you may also use this fact to put a quadratic in standard form.)

Section 3.2. Graphs of polynomials: Graphing $y = x^n$ and transformations. Leading coefficients and right-hand and left-hand behaviors. Zeros, factors, and x-intercepts. Graph-sketching.

Section 3.3. Long division of polynomials: Long procedure; synthetic division for dividing by $(x - c)$. Format of answer: $P(x) = D(x)Q(x) + R(x)$ or $\dfrac{P(x)}{D(x)} = Q(x) + \dfrac{R(x)}{D(x)}$. Remainder and factor theorems.

Section 3.4. Rational zeros theorem, with applications and examples. Finding real zeros. Procedure for finding rational or real zeros of a polynomial: Try zeros, divide when possible, keep trying until reduced to quadratic. *Not tested:* Intermediate value theorem, zeros via graphing calculator.

Section 3.5. NOT ON FINAL.

Section 3.6. NOT ON FINAL.

Section 3.7. NOT ON FINAL.

Section 4.1. Idea of inverse function; definition of inverse function. Checking $f(x)$ and $g(x)$ are inverses. Finding inverse functions. Graphs of inverse functions horizontal line test.

Section 4.2. Definition of exponential function with base a. Graphs of exponential functions a^x, a^{-x} $(a > 1)$; domain, range, horizontal asymptote, key points. The natural base e. Compound interest.

Section 4.3. Definition of $\log_a x$, logarithmic function with base a. Properties of logarithms (based on definition). Graphs of logarithmic functions, $a > 1$; domain, range, vertical asymptote, key points. Natural log $\ln x$.

Section 4.4. Change of base formula. Log rules: $\log_a(uv)$, $\log_a(u/v)$, $\log_a(u^n)$. Expanding and combining log expressions.

Section 4.5. Solving exponential equations: Standard types, tricky types. Solving logarithmic equations.

Section 4.6. NOT ON FINAL.

Section 5.1. Systems of two equations: Solving by substitution. No solutions, multiple solutions. Graphical approach. Applications.

Section 5.2. Systems of two linear equations: Solving by elimination. Graphical interpretation of solutions. Applications.

Section 5.3. Systems of three or more linear equations: Row-echelon form and back-substitution. Gaussian elimination. (*Not on final:* Numbers of solutions: Zero, one, infinity. Nonsquare systems. Applications.)