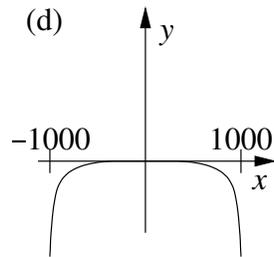
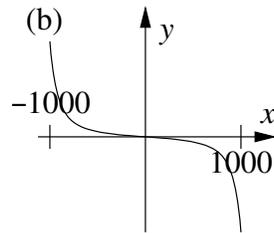
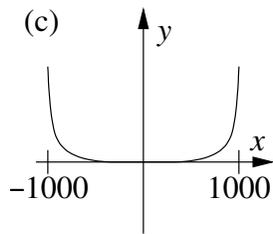
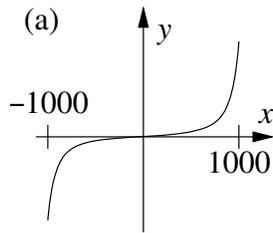


**Sample Final Exam**  
**Math 18A, Spring 2024**

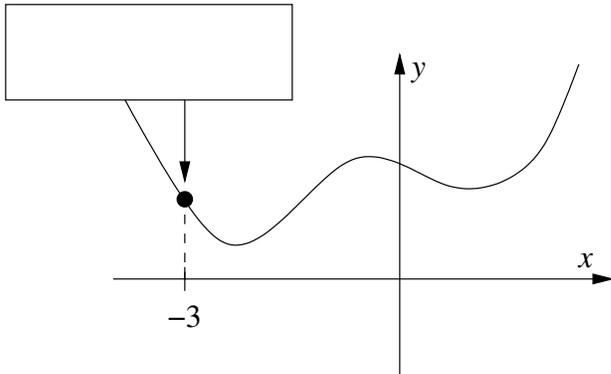
1. (7 points) Find two functions  $f$  and  $g$  such that  $(g \circ f)(x) = \sqrt[3]{\frac{1}{1+x^2}}$  and neither  $g(x) = x$  nor  $f(x) = x$ . No explanation necessary.
2. (7 points) Solve the inequality  $7 - 3x < 12$ . You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).
3. (7 points) Find all **real** solutions to the equation  $2x^2 + 5x - 3 = 0$ . If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).
4. (7 points) Consider the function  $f(x) = -6x^9 + 129x^4$ . Which of the graphs below best matches the graph of  $f(x)$ ? Note that the horizontal scale on the graph goes from  $x = -1000$  to  $x = 1000$ , and the vertical scale is unspecified. Circle your answer, and briefly **EXPLAIN** how the **TWO** most important features of the coefficients and powers appearing in  $f(x)$  justify your conclusion.



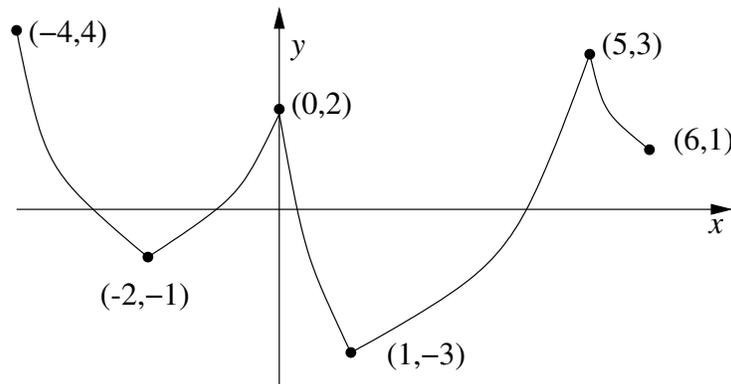
5. (7 points) The graph below (not to scale) is a sketch of the graph of the function

$$f(x) = x^4 - 8x^2 + 3x + 387.$$

Fill in the coordinates for the indicated point. Show all your work.



6. (7 points) Suppose  $f$  is a one-to-one function. If we know that  $f(7) = -4$ , what can we say about  $f^{-1}$ ? No explanation necessary.
7. (9 points) Suppose  $f(x)$  is a function whose graph is shown below (not to scale).

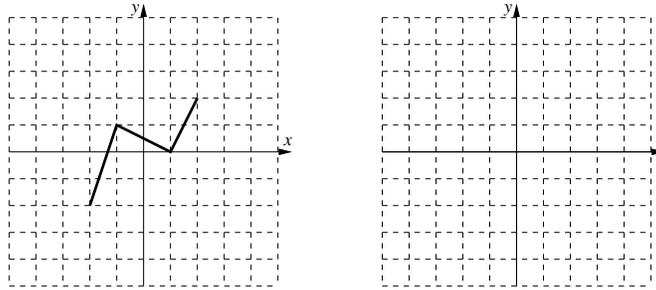


For the following, you may express interval answers either in interval notation (e.g., “on the intervals  $(22, 55)$  and  $[-16, 3]$ ”) or by inequalities (e.g., “for  $x > 325$ ”).

- (a) On which interval or intervals (values of  $x$ ) is the function  $f(x)$  **increasing**?
- (b) At which **value(s) of  $x$**  does  $f(x)$  have a **relative maximum**?
8. (9 points) Draw the graph of  $g(x) = 2310^x$ . Clearly label any  $x$ -intercepts,  $y$ -intercepts, or asymptotes.
9. (9 points) Expand the expression  $\log\left(\frac{(x-7)^5 \sqrt{x+2}}{(x-4)^3}\right)$  as a sum, difference, and/or multiple of logarithms. Show all your work.

10. (9 points) Find all solutions to the equation  $\frac{3}{x-4} + \frac{5}{x-2} = 2$ . **USE ALGEBRA**, not guessing, to determine your answer. Show all your work, and leave your answer(s) in exact form (i.e., radicals and fractions, not decimals).

11. (9 points) Let  $y = f(x)$  be the function whose graph is shown below left. (Each square is 1 unit  $\times$  1 unit.) On the axes below right, graph the function  $y = f(x + 1) + 2$ , paying careful attention to the vertical and horizontal scales. No explanation necessary.



12. (9 points) Find the equation of the line through the points  $(-3, -12)$  and  $(10, 7)$ . Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

13. (12 points) Let  $f(x) = -x^2 + 2x$ . Simplify  $\frac{f(5+h) - f(5)}{h}$  completely. Show all your work.

14. (12 points) Find all possible solutions to the following system:

$$\begin{aligned} 7x + 4y &= 3, \\ 2x - 3y &= 2. \end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

15. (12 points) Show all your work in both parts of this question.

- Write the quadratic function  $f(x) = x^2 + 36x - 523$  in vertex form (also known as standard form).
- Graph the quadratic function  $f(x) = x^2 + 36x - 523$ . Label the vertex and the  $y$ -intercept of your graph. (You do not need to label the  $x$ -intercept(s), if any.)

16. (12 points) Find the domain of the function  $g(x) = \frac{\sqrt{4-x^2}}{x+1}$ . Show all your work. Express your answer in **interval notation** (e.g.,  $[-16, 3] \cup (22, 55)$ ).

**17.** (14 points) Pop star Traylor Switt and NFL linebacker Tavis Kelsey are holding a one-day charity benefit sale. Switt is selling copies of her album *1987 (the Prime version)* for \$23 each and Kelsey is selling replica football jerseys for \$35 each. They sell a total of 11,883 items (albums and jerseys together) and take in a total of \$325,533 in revenue.

How many *1987 (the Prime version)* albums did they sell during the charity benefit sale?

**USE ALGEBRA**, not guessing, to determine your answer. Show all your work and give your final answer in the form of a complete sentence, using the correct units and rounding off the numerical part of your answer to the nearest whole number.

**18.** (14 points) Let

$$f(x) = x^3 + 10x^2 + 31x + 30.$$

Find the rational zeros of  $f(x)$  by factoring  $f(x)$  completely. Show all your work. Make sure you include both the complete list of zeros of  $f(x)$  and the factorization of  $f(x)$  in your final answer.

**19.** (14 points) The long-running British science fiction show *Doctor Who???* has had a recent run of success, and membership in the *Doctor Who???* fan club has risen quite a bit since its low point of 250,000 members at the beginning of 2015. In fact, if  $P(t)$  is the number of members of the *Doctor Who???* fan club  $t$  months after the beginning of 2015, then  $P(t)$  can be modelled by

$$P(t) = 250000e^{0.1t}.$$

Assuming that trend has continued, how many months after the beginning of 2015 did it take for membership in the *Doctor Who???* fan club to reach 400,000?

**USE ALGEBRA**, not guessing or calculator estimation, to determine the answer. Show all your work, round off your final numerical answer to the nearest .1, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

**20.** (14 points) Consider the polynomial function

$$f(x) = (x - 6)(x - 7)(x - 8)(x - 71)(x - 72)(x + 999)(x + 1000)(x + 1001).$$

- List the real zero(s) of  $f$ .
- Sketch the graph of  $f(x)$ . You do not have to draw your graph to scale, but make sure that the above information you found about zeros is clearly visible in your graph.