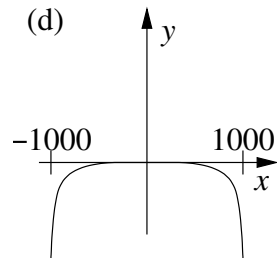
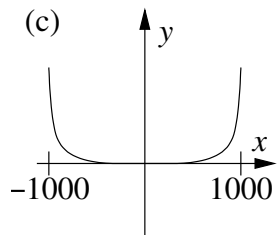
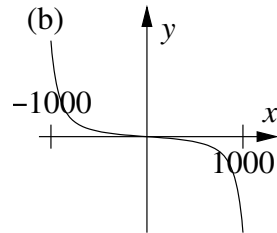
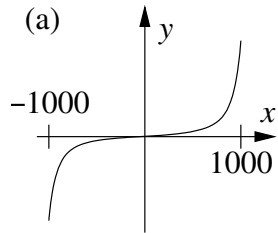
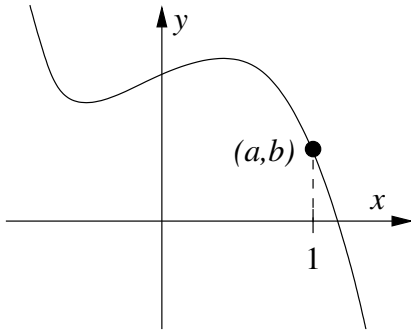


**Sample Final Exam**  
**Math 18A, Fall 2020**

1. (7 points) Consider the function  $f(x) = 8x^{27} - 13x^{10}$ . Which of the graphs below best matches the graph of  $f(x)$ ? Note that the horizontal scale on the graph goes from  $x = -1000$  to  $x = 1000$ , and the vertical scale is unspecified. **CLEARLY** indicate your answer, (a), (b), (c), or (d), and briefly **EXPLAIN** how the **TWO** most important features of the coefficients and powers appearing in  $f(x)$  justify your conclusion.



2. (7 points) Below is a sketch of the graph of the function  $f(x) = 10x^3 - 23x + 150$  (not to scale). Find the coordinates of the point  $(a, b)$  (i.e., solve for  $a$  and  $b$ ). Show all your work.



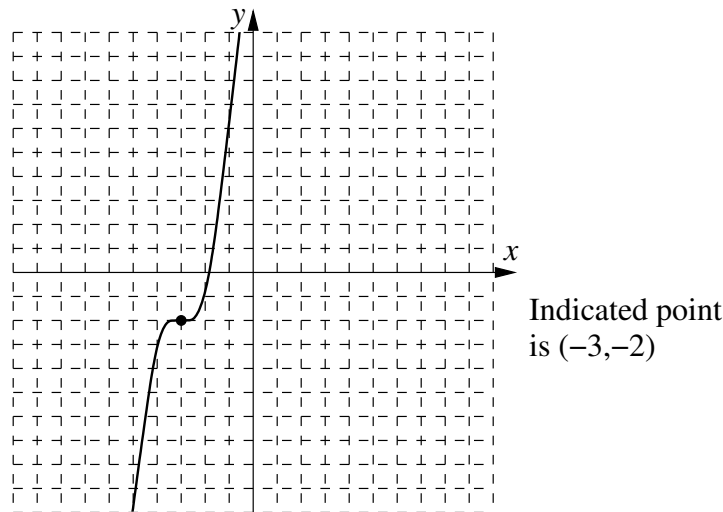
3. (7 points) Find two functions  $f$  and  $g$  such that  $(g \circ f)(x) = \sqrt{x^2 + 3}$  and neither  $g(x) = x$  nor  $f(x) = x$ . No explanation necessary.
4. (7 points) Solve the inequality  $0 < 3x + 5 < 12$ . You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).
5. (7 points) Suppose  $f$  is a one-to-one function. If we know that  $f(-2) = 5$ , what can we say about  $f^{-1}$ ? No explanation necessary.

6. (7 points) Find all **real** solutions to the equation  $x^2 - 13x + 15 = 0$ . If there are no real solutions, briefly (1 sentence) **EXPLAIN** how you know there are no real solutions. Show all your work, and leave your answer(s) (if any) in exact form (i.e., radicals and fractions, not decimals).

7. (9 points) Find the equation of the line through the points  $(2, -7)$  and  $(5, -17)$ . Show all your work, and leave the numbers in your final answer in fractional form (not decimals). (You will receive full credit for a correct answer left in point-slope form.)

8. (9 points) Expand the expression  $\ln\left(\frac{(x-7)^2\sqrt{x+11}}{(x+3)^5}\right)$  as a sum, difference, and/or multiple of logarithms. Show all your work.

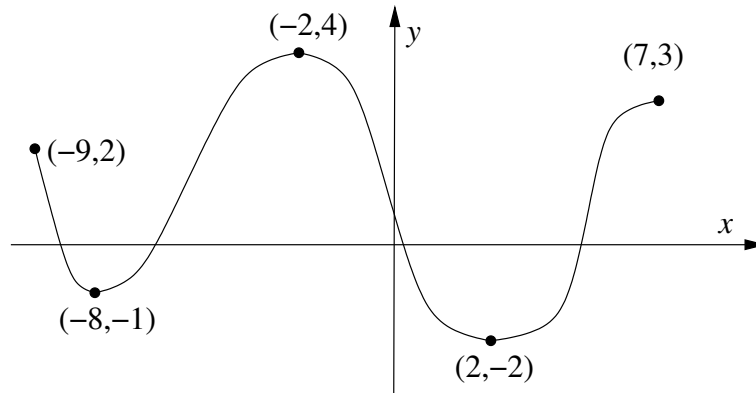
9. (9 points) Use the graph of  $f(x) = x^3$  to write an equation for the function whose graph is shown below. (Each square in the graph is 1 unit  $\times$  1 unit, and the indicated point is  $(-3, -2)$ .)



10. (9 points) Draw the graph of  $g(x) = \left(\frac{1}{753}\right)^x$ . Clearly label any  $x$ -intercepts,  $y$ -intercepts, or asymptotes. You do not need to draw your graph to scale, but make sure that the key features of your graph (e.g., intercepts and asymptotes) are clearly visible.

11. (9 points) Find all solutions to the equation  $\frac{3}{x+1} - \frac{3}{x-5} = 2$ . **USE ALGEBRA**, not guessing, to determine your answer. Show all your work, and leave your answer(s) in exact form (i.e., radicals and fractions, not decimals).

12. (9 points) Suppose  $f(x)$  is a function whose graph is shown below (not to scale).



- (a) On which interval or intervals (values of  $x$ ) is the function  $f(x)$  **decreasing**? You may express your answer either in interval notation (e.g., “on the intervals  $(22, 55)$  and  $[-16, 3]$ ”) or by inequalities (e.g., “for  $x > 325$ ”).
- (b) At which **value(s)** of  $x$  does  $f(x)$  have a **relative maximum**?

13. (12 points) Find the domain of the function  $g(x) = \frac{\sqrt{x^2 - 25}}{x + 13}$ . Show all your work. You may express your answer either in interval notation (e.g., “ $[-16, 3] \cup (22, 55)$ ”) or by inequalities (e.g., “ $x > 325$ ”).

14. (12 points) Let  $f(x) = x^2 + 5x$ . Simplify  $\frac{f(3+h) - f(3)}{h}$  completely. Show all your work.

15. (12 points) Write the quadratic function  $f(x) = x^2 - 444x + 175000$  in standard form and sketch its graph. Choose your scale and axes so that all important points are visible, and label the vertex and the  $y$ -intercept of your graph. (You do not need to label the  $x$ -intercept(s), if any.) Show all your work.

16. (12 points) Find all possible solutions to the following system:

$$\begin{aligned}2x + 7y &= -1, \\3x - 5y &= 4.\end{aligned}$$

If there are no solutions, or infinitely many solutions, briefly **EXPLAIN** how you know this is true. Show all your work, and leave all numerical answers in exact form (fractions, radicals, etc.). Note that solutions need not be whole numbers.

17. (14 points) Let

$$f(x) = x^3 + 7x^2 + 14x + 8.$$

Find the rational zeros of  $f(x)$  by factoring  $f(x)$  completely. Show all your work. Make sure you include both the complete list of zeros of  $f(x)$  and the factorization of  $f(x)$  in your final answer.

**18.** (14 points) Geoff Zebos, the second richest person in the world, has seen a great increase in his personal wealth since the beginning of April. At the beginning of April, his personal wealth stood at \$113.2 billion, and  $t$  days after the beginning of the year, his personal wealth was

$$P(t) = 113.2e^{.003t}$$

(in billions of dollars). How many days after the beginning of April did it take for Zebos' wealth to reach \$150 billion?

**USE ALGEBRA**, not guessing or calculator estimation, to determine the answer. Show all your work, round off your final numerical answer to the nearest .1, if necessary, and give your final answer in the form of a complete sentence, using the correct units.

**19.** (14 points) Consider the polynomial function

$$f(x) = (x - 11)(x - 12)(x - 13)(x + 20)(x + 21)(x - 52)(x - 53).$$

- (a) List the real zero(s) of  $f$ .
- (b) Sketch the graph of  $f(x)$ . Make sure that the key qualitative features of the graph are clearly visible in your graph, including the above information you found about zeros. You do not have to draw your graph to scale; in fact, it may not be possible to draw the graph to scale and also make the key features of the graph clearly visible.

**20.** (14 points) Minor crime boss Ann German Agement has two sources of income, extortion and illegal gambling. In 2019, she made a total of \$3,956,202, and she made 25% more money from illegal gambling than she did from extortion.

How much money did Ann made from illegal gambling in 2019?

**USE ALGEBRA**, not guessing, to determine your answer. Show all your work and give your final answer in the form of a complete sentence, using the correct units, rounding off the numerical part of your answer to the nearest .01.