Math 145, Spring 2001 **Name:** ______________________

Exam 2

Due: Wed Apr 25

This test consists of 5 questions on 10 pages, totalling 100 points. You may use anything we have covered in the class notes and the class textbook freely, but you cannot receive help from other people. More specifically, unless otherwise stated, you may take as given anything that has been proven in class, in the homework, or in the reading.

1. (15 points) Let S be the surface obtained by identifying opposite sides of an 18-gon in an orientation-preserving manner, i.e., the side-paired 18-gon with defining relation $a_1 a_2 \ldots a_9 a_1^{-1} a_2^{-1} \ldots a_9^{-1}$. Find, with proof, either a connected sum $T^2 \# \cdots \# T^2$ or a connected sum $P^2 \# \cdots \# P^2$ that is homeomorphic to S, and specify how may tori or projective planes are in the sum. You may assume all of the results from the classification of surfaces (Notes Chap. 7).

(Problem continued on next page.)

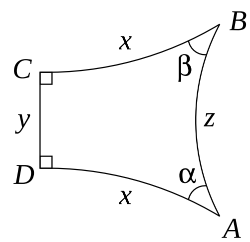
(Extra space for problem 1.)

2. (15 points) Let $A = (0,0)$, $B = (1,1)$, $C = (0,2)$, $f = \gamma_{AB}$, $g = \gamma_{AC}$. Describe the Euclidean isometry gfg^{-1} in standard form (i.e., as a translation, rotation, reflection, or glide-reflection).

(Problem continued on next page.)

(Extra space for problem 2.)

3. (20 points) Consider the hyperbolic quadrilateral $ABCD$ (in counterclockwise order) with $h(C, D) = y$, $h(A, B) = z$, $h(B, C) = h(D, A) = x$, and right angles at vertices C and D, as shown below.



(a) Prove that $\alpha = \beta$.

(b) Prove that $\alpha < \pi/2$.

(c) Prove that $z > y$.

(Suggestions: You may find it helpful to use trigonometry, standard position, isometries, and other triangle theorems.)

(Problem continued on next page.)

(Extra space for problem 3.)

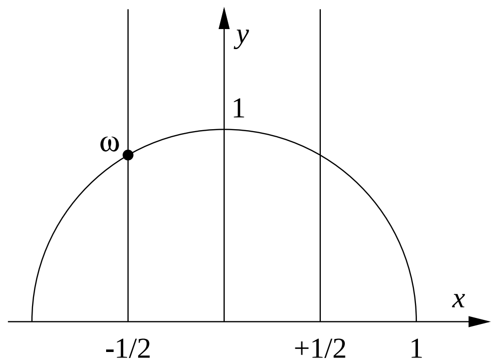
4. (20 points) Recall that for $f \in \text{Isom}(\mathbf{H}^2)$, the *minimal motion* of f, denoted by $\mu(f)$, is the infimum of $h(x, f(x))$ over all $x \in \mathbf{H}^2$, and the *set of minimal motion* of f is the set of all $x \in \mathbf{H}^2$ such that $h(x, f(x)) = \mu(f)$.

Let $f \in \text{Isom}(\mathbf{H}^2)$ be an isometry of hyperbolic type. Prove that there is a unique geodesic q such that $f(q) = q$, setwise, and prove that q is the set of minimal motion of f. (You may use the results of question 3, even if you do not complete it.)

(Problem continued on next page.)

(Extra space for problem 4.)

5. (30 points) The picture of $\mathbf{H}^2$ below shows the lines L_1 ($x = -1/2$) and L_2 ($x = +1/2$) and the Euclidean unit circle. The complex value of the indicated point is $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$.



(a) Find a parabolic $p \in \mathrm{Isom}(\mathbf{H}^2)$ such that $p(L_1) = p(L_2)$.

(b) Suppose $f \in \mathrm{Isom}(\mathbf{H}^2)$ is an orientation-preserving isometry such that the order of f is 3. Prove that $\tau(f) = 1$.

(c) Find an orientation-preserving $f \in \mathrm{Isom}(\mathbf{H}^2)$ such that $f(\omega) = \omega$ and the order of f is 3. (Suggestion: Normalize f to make $\det(f) = 1$ (how do you know you can do this?).)

(Problem continued on next page.)

(Extra space for problem 5.)