Sample Exam 2
Math 42, Fall 2014

This is most of the second exam I gave in Math 42 in Spring 2014. Our exam may be more difficult than this one, but it should at least be comparable, and cover similar (though not the same) material. Exception: Our exam will cover the additional topics of algorithms, recursion, and induction, as well as sections 6.4–6.5.

1. (10 points) Let G be a graph.

(a) Define the **length** of a path in G.

(b) Define the **distance** between two vertices v, w in G.

2. (16 points) **INDUCTION SETUP.** Set up, but **DO NOT COMPLETE**, the proof of the following theorem by induction on n:

> **Theorem.** For $n \geq 3$, the sum of the interior angles of a convex polygon with n sides is $(n-2) \cdot 180°$.

You may set up either a proof by standard induction or a proof using strong induction.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z				
15	16	17	18	19	20	21	22	23	24	25				

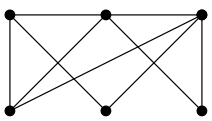
3. (10 points) Suppose the ciphertext MPGGF has been encoded by a +7 shift cipher. Decode the message, and circle the (decoded) plaintext. No explanation necessary, but show all your work.

4. (10 points) Encode the plaintext MXYZPTLK using the standard Vignére cipher and the key word CKENT. No explanation necessary, but show all your work.

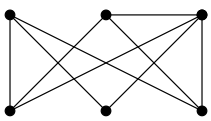
5. (10 points) Nikita is ordering veggie stir-fry at the Random Wok lunch counter. For one order of stir-fry, Nikita needs to choose 3 types of vegetables from among 5 possibilities, and she needs to choose 4 sauce ingredients from among 9 possibilities.

How many possible stir-fry orders can Nikita make? Briefly (1–2 sentences) **JUSTIFY** your answer, and leave your final answer in terms of choice notation. (E.g., you do not need to calculate any factorials; in fact, you do not even need to write them down.)

6. (14 points) Consider the graphs G_1 and G_2 drawn below. Are G_1 and G_2 isomorphic? Briefly **JUSTIFY** your answer.



G_1



G_2

7. (16 points) Use induction to prove that

$$\sum_{j=1}^{n}(3j-1) = \frac{n(3n+1)}{2}$$

for $n \geq 1$.