Sample Final Exam
Math 42, Fall 2014

This is most of the final I gave in Math 42 in Spring 2014. Our exam may be more difficult than this one, but it should at least be comparable, and cover similar (though not the same) material. Exceptions are as noted on all four review sheets; most notably, Ch. 11 was not on last semester's final, but will be covered on this semester's final.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
P	Q	R	S	T	U	V	W	X	Y	Z				
15	16	17	18	19	20	21	22	23	24	25				

1. (10 points) Encode the message YELLOW using a +10 shift code (see above for letter-number translations). Show all your work.

2. (10 points) In a Mardi Gras parade, the Mixt Motivez Krewe is handing out identical beaded necklaces. When they throw necklaces into a crowd, they end up being distributed arbitrarily; no one is guaranteed to get a necklace, and conversely, there is no limit on the number of necklaces one person can get. If they throw 25 necklaces into a crowd of 41 people, in how many different ways can the necklaces end up distributed?

Show all your work, and briefly **JUSTIFY** your answer. In particular, you should not just have numbers; you should give some indication of the method you are using, and how it applies here.

3. (10 points) Write the negation of the following statement, **without using the word "not":** "There is a place in this world where everybody knows your name."

4. (12 points) Suppose P, Q, and R are logical statements, and the statement

$$(P \wedge Q) \Rightarrow R$$

is FALSE. What (if anything) can you deduce about the truth or falsity of P, Q, and R? Briefly (2–3 sentences) **EXPLAIN** your answer.

5. (12 points) Let $A = \{1, 3\}$, $B = \{2, 4, 6, 8\}$, and $C = \{7, 8, 9\}$. List the elements of $A \times (B \backslash C)$. No explanation necessary, but show all your work.

6. (12 points) At the restaurant *Awffle House*, hash browns are served either plain, with melted cheese, with grilled onions, or with both cheese and onions. A careful study of *Awffle House* customers shows that $\frac{3}{5}$ of all orders of hash browns include cheese, $\frac{1}{5}$ of all orders include both cheese and grilled onions, and $\frac{3}{10}$ of all orders are plain. If a random customer orders hash browns with onions, what is the probability that they will also include cheese?

Show all your work, and briefly **JUSTIFY** your answer. In particular, you should not just have numbers; you should give some indication of the methods you are using, and how they apply here.

7. (12 points) Codex often plays the online game *Whirled Ol' Warcruft.* During the day, she plays with one team of 6 people (not including her), the Day Troopers, and at night, she plays with a different team of 5 people (not including her), the Nighty-Knights. However, because of bad incidents in the past, all of the other Day Troopers refuse to play with the Nighty-Knights, and vice versa.

One day, she needs to choose 3 other players for a raid. The raid is at twilight, so she can play with people from either team, but the team cannot include players from both teams at the same time (see above). So, if she needs to choose either 3 Day Troopers or 3 Nighty-Knights, but not both (and certainly no mixing of the two teams), in how many ways can she choose those 3 players?

Show all your work, and briefly **JUSTIFY** your answer. In particular, you should not just have numbers; you should give some indication of the methods you are using, and how they apply here.

8. (12 points) Consider a sequence a_n that begins $-5, 0, 9, 22, 39, 60, 85, \ldots,$ where $a_0 = -5$, $a_1 = 0$, and so on. Assuming this pattern continues, find a closed form for a_n. Show all your work.

9. (12 points) Jennifer and David are handing out cookies at a children's birthday party. They have 7 different types of regular cookies to hand out and 4 types of vegan cookies to hand out. Suppose that there are 7 children at the party, 2 of whom can only eat the vegan cookies, and the other 5 of whom can eat either vegan or regular cookies. Assuming that Jennifer and David have an effectively unlimited supply of each type of cookie, and assuming that all cookies of the same type are identical, how many ways are there for them to give each child **exactly one** cookie?

Show all your work, and briefly **JUSTIFY** your answer. In particular, you should not just have numbers; you should give some indication of the methods you are using, and how they apply here.

10. (12 points) Consider the graphs G_1 and G_2 drawn below. Briefly **EXPLAIN** how you can be sure that G_1 and G_2 are **not** isomorphic.

G_1 G_2

11. (14 points) Let $f : \mathbb{Z} \to \mathbb{R}$ be the function with domain $\mathbb{Z}$ and target space $\mathbb{R}$ given by the formula $f(x) = 2x$. Briefly **EXPLAIN** why f is **not** onto.

12. (14 points) Let S be the set of all people who are on campus right now at SJSU, let $D = \{\text{Jan 1}, \text{Jan 2}, \ldots, \text{Dec 31}\}$ be the set of all days of the year (including Feb 29), and let $B : S \to D$ be the function given by the formula

$$B(s) = \text{the birthday of } s$$

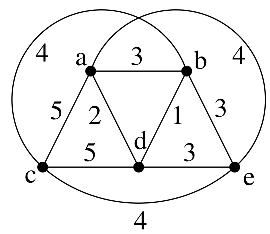
for all $s \in S$. (For example, $B(\text{Tim Hsu}) = \text{Feb 18}$.)

Use the Pigeonhole Principle to **EXPLAIN** briefly why B is **not** one-to-one.

13. (16 points) The Gamble-O-Tron is a machine that contains 37 red balls, 10 yellow balls, and 3 green balls. When you put \$1 into the Gamble-O-Tron, a randomly chosen ball comes out. If you get a red ball, you win nothing; if you get a yellow ball, you win \$5; and if you get a green ball, you win \$10.

What is the expected value of the amount of money you win or lose by playing the Gamble-O-Tron once, including the \$1 you are sure to lose by playing? Show all your work, and express your final answer in the form of a complete sentence, using the correct units, and making clear whether you should expect to win or lose money by playing.

14. (16 points) Consider the following weighted graph G:



Use either Kruskal's algorithm or Prim's algorithm to find a minimum weight spanning tree for G, and find the weight of that spanning tree. Show all your work; specifically, describe or draw what happens each time you add an edge. You may refer to an edge by the two vertices it touches; for example, the edge bd is the unique edge labelled 1.

15. (16 points) Use induction to prove that

$$\sum_{j=0}^{n}(2j+3) = n^2 + 4n + 3$$

for $n \geq 0$. (Note: Since the point of this problem is to see if you can set up and complete an induction proof correctly, proving this formula without using induction will receive very little credit.)