Topics for Exam 3
Math 42, Fall 2014

General information. Exam 1 will be a timed test of 75 minutes, covering 6.1–6.9, 7.1–7.6, 8.1–8.9, and 10.1–10.2 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will be based on the homework and quizzes from the above sections. If you can do all of those problems, and you know and understand all of the ideas behind them, you should be in good shape.

As mentioned above, your first priority should be to understand the homework and quizzes and the ideas behind them. Besides the list of topics you should know, below, you should also be familiar with everything specially emphasized in the text. You should also study the Check Yourself problems in 6.2, 6.4, 6.6, 6.7, 6.8, 7.2, 7.4, 7.5, 8.2, 8.3, 8.5, 8.6, and 10.2. These are practice problems with answers in the back of the book, and similar problems may well appear on the exam. Finally, you should also look at the Try This problems in 6.3, 6.6, 6.9, 7.3, 7.6, 8.4, 8.7, and 8.9; these do not have answers in the back of the book, but you are welcome to ask about them in class and at office hours.

Statements of definitions. On at least one question, you will be asked to recite one of the definitions listed below (the italicized words under the **Definitions:** headings). This is meant to force you to learn the mathematical terms that you need to learn for the rest of this course and for future courses.

Section 6.2. Definitions: $\binom{n}{k}$, *n choose k, recursion.* Choice notation identity (p. 156).

Section 6.3–6.4. Pascal's Triangle; computing $\binom{n}{k}$ using triangle/choice notation identity.

Section 6.5. Definitions: *k-to-one correspondence,* $n!$ *(n factorial).*

Section 6.7. Definitions: *Binomial, binomial coefficients, binomial identity.* Binomial Theorem, Factorial formula for $\binom{n}{k}$.

Section 6.8. Definitions: *Combinatorial proof, bijective proof.* Examples of combinatorial proofs.

Section 7.2. Definitions: *Labelled vs. identical balls, with/without repetition.* See balls in boxes review, below.

Section 7.4. Solutions to balls in boxes problems (see review, below). Strong suggestion: Don't memorize formulas, remember a strategy/story for each type of problem.

Section 7.5. PIE (Principle of Inclusion/Exclusion): 2 sets case, 3 sets case.

Section 8.2. Fibonacci numbers: Definition, examples of identities, proving identities by induction and otherwise.

Section 8.3. Definitions: *Closed form, explicit formula; recurrence, recursion.* Proving closed forms and other identities by induction.

Section 8.5. Naive methods (i.e., guessing) for closed forms.

Section 8.6. Closed forms for recurrences of the form $a_n = a_{n-1} + p_k(n)$, where $p_k(n)$ is a degree k polynomial in the variable n.

Section 8.8. Closed forms for linear homogeneous recurrences with constant coefficients.

Section 10.2. Definitions: *connected, cycle, tree.* Ex. 4.2.5: If a tree has n vertices, then it has $n-1$ edges. Partial converses to Ex. 4.2.5 (Thms. 10.2.1, 10.2.2; Cor. 10.2.3). Theorems about leaves (10.2.4, 10.2.5).

Not on exam: Sections 6.11–6.12, 7.8, 8.10–8.11.

Review of balls in boxes

Type	How many ways are there to put	with	into n labelled boxes?
A	k labelled balls	≤ 1 per box	$n(n-1)\cdots(n-(k-1))$
B	k identical/unlabelled balls	≤ 1 per box	$\binom{n}{k}$
C	k types of balls	exactly 1 per box	k^n
D	k identical/unlabelled balls	no limit per box	$\binom{n-1+k}{k} = \binom{n-1+k}{n-1}$
D′	k identical/unlabelled balls	≥ 1 per box	First 1 ball in each box, then problem D
D*	k labelled balls	no limit per box	n^k
E	k labelled balls	k_i in box i, $k_1+\cdots+k_n = k$	$\frac{k!}{k_1!k_2!\ldots k_n!}$

And there's also problem type F: How many anagrams of a given word are there? (Solved similarly to problem type E.)

Of course, it's better to remember stories and not just formulas:

- A: Put one ball in at a time, avoiding previously filled boxes.
- B: Choose which k boxes get balls.
- C: Choose which type of ball goes in each box, n times.
- D, D′: Stars and bars.
- D*: Choose which box to put each ball in, k times.
- E: Each ordering of the k balls determines which balls go in which boxes, overcount by the orderings of the balls within each box.