Format and topics for exam 3
Math 42

General information. Exam 3 will be a timed test of 75 minutes, covering sections 4.1–4.3, 5.1–5.2, 6.1–6.3, and 7.1–7.2 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, but you should definitely spend time memorizing the *statements* of the important results in the text, especially any result with a name.

Types of questions. Exam 3 will feature the same potential types of questions as the previous exams: Statements of definitions and theorems, computations, and problem-solving with explanation.

Definitions. The most important definitions and symbols we have covered are:

4.1	a divides b	divisor, factor
	divisor	quotient
	remainder	div
	mod	a congruent to b mod m
	$a \equiv b \pmod{m}$	congruence
	modulus	$\mathbf{Z}_m$
4.2	base b expansion	decimal
	binary	octal
	hexadecimal	
4.3	prime	composite
	greatest common divisor	relatively prime
	pairwise relatively prime	least common multiple
5.1	basis step	inductive step
6.3	permutation	r-permutation
	r-combination	binomial coefficient $\binom{n}{k}$
	combinatorial proof	
7.1	experiment	sample space
	event	discrete probability $\lvert E\rvert \,/\, \lvert S\rvert$
7.2	probability distribution	uniform distribution
	probability of an event E	at random, randomly
	conditional probability	independent (events)
	pairwise independent	mutually independent

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to cite them as needed. You should also be prepared to recite named theorems.

Sect. 4.1: Properties of divisibility (Thm. 1, Cor. 1). The Division Algorithm (Thm. 2). Mod is mod (Thm. 3), mod is remainder (Thm. 4). Mod preserves addition and multiplication (Thm. 5, Cor. 2).

Sect. 4.2: Base b expansion exists and is unique (Thm. 1).

Sect. 4.3: Fundamental Theorem of Arithmetic (prime factorization). $ab = \gcd(a, b)\,\mathrm{lcm}(a, b)$.

Sect. 5.1: Principle of Mathematical Induction.

Sect. 5.2: Principle of Strong Induction.

Sect. 6.1: Product rule (AND), sum rule (XOR). Subtraction rule (inclusion-exclusion). Division rule.

Sect. 6.2: Pigeonhole principle (Thm. 1, Cor. 1). Generalized pigeonhole (Thm. 2).

Sect. 6.3: Formulas for $P(n, r)$ (Thm. 1, Cor. 1), $C(n, r)$ (Thm. 2). Symmetry: $C(n, r) = C(n, n - r)$ (Cor. 2).

Sect. 7.1: Probability of complement (Thm. 1), union (Thm. 2).

Sect. 7.2: Probability of pairwise disjoint union (Thm. 1).

Types of problems. You should also know how to do the following general types of problems, some of which are straight computations, and some of which require explanation. (Note also that on the actual exam, there may be problems that are not one of these types. Nevertheless, it will be helpful to know how to do all these types.)

Sect. 4.1: Applications of the Division Algorithm. Quotient and remainder, **div** and **mod**; mod m calculations. Applications: clocks, day of the week, etc.

Sect. 4.2: Base conversions among decimal, binary, octal, and hexadecimal..

Sect. 4.3: Find the prime factorization of a given n. Which integers are relatively prime to n? GCD and LCM in terms of prime factorization.

Sect. 5.1: Induction proofs: Summation formulas, inequalities, set identities.

Sect. 5.2: Strong induction proofs: Stamp problem, binary representation. Variations on strong induction.

Sect. 6.1: Multiplication and addition principles, with constraints: Multiple choices, bit strings/random words, decimal numbers with certain digits.

Sect. 6.2: Pigeonhole principle: Straightforward applications.

Sect. 6.3: Permutations and combinations with constraints, combined with multiplication and addition principles.

Sect. 7.1: Using Ch. 6 to count events: playing cards, lottery, dice.

Sect. 7.2: More counting probabilities: Dice, permutations, bitstrings. Are events independent?

Not on exam. Subsections 4.2.3–4.2.4, 4.3.3–4.3.5, and 4.3.7–4.3.8. Subsection 5.2.5. Subsections 6.1.6 and 6.2.3. Subsections 7.2.7–7.2.10.