

Format and topics for exam 2
Math 42

General information. Exam 2 will be a timed test of 75 minutes, covering sections 1.7–1.8 and 2.1–2.4 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we’ve studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, but you should definitely spend time memorizing the *statements* of the important results in the text, especially any result with a name (e.g., DeMorgan’s Laws for set operations).

Types of questions. Exam 2 will feature the same potential types of questions as Exam 1: Statements of definitions and theorems, computations, and problem-solving with explanation.

Definitions. The most important definitions and symbols we have covered are:

1.7	theorem proof lemma conjecture even, odd perfect square rational number proof by contraposition counterexample circular reasoning	proposition axiom corollary direct proof same parity indirect proof irrational number proof by contradiction begging the question
1.8	proof by cases perfect power existence proof nonconstructive proof arithmetic mean	proof by exhaustion without loss of generality constructive proof uniqueness proof geometric mean
2.1	set roster method natural numbers \mathbb{N} positive integers \mathbb{Z}^+ real numbers \mathbb{R} complex numbers \mathbb{C} open interval (a, b) empty set \emptyset universal set set equality $A = B$ proper subset, $A \subset B$ infinite set ordered n -tuple Cartesian product $A \times B$ relation	$x \in A, x \notin A$ set-builder notation integers \mathbb{Z} rational numbers \mathbb{Q} positive real numbers \mathbb{R}^+ interval closed interval $[a, b]$ singleton set Venn diagram subset, $A \subseteq B$ finite set power set $\mathcal{P}(S)$ ordered pair $A_1 \times A_2 \times \cdots \times A_n$ truth set
2.2	union $A \cup B$ disjoint sets universe U membership table	intersection $A \cap B$ set difference $A - B$ complement \bar{A}

2.3	function	$f : A \rightarrow B$
	domain	codomain
	image	preimage
	range	equal (functions)
	real-valued (function)	integer-valued (function)
	one-to-one	injection
	increasing	strictly increasing
	decreasing	strictly decreasing
	onto	surjection
	one-to-one correspondence	bijection
	identity function	inverse function
	invertible	composition
	floor function $\lfloor x \rfloor$	ceiling function $\lceil x \rceil$
	factorial function $n!$	
2.4	sequence	term
	geometric sequence	initial term (of a geom seq)
	common ratio (of a geom seq)	arithmetic progression
	initial term (of an arithm prog)	common difference (of an arithm prog)
	recurrence relation	solution (of a recurrence)
	initial conditions	Fibonacci sequence
	closed formula	(solution by) iteration
	Lucas sequence	summation notation
	index of summation	lower/upper limit of sum
	geometric series	

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to cite them as needed. You should also be prepared to recite named theorems.

Sect. 2.1: Empty set is a subset of every other set; every set is a subset of itself.

Sect. 2.2: Set identities (Table 1), especially distributive law and DeMorgan's Laws for sets.

Sect. 2.4: Sum of a geometric series (Theorem 1).

Types of problems. You should also know how to do the following general types of problems, some of which are straight computations, and some of which require explanation. (Note also that on the actual exam, there may be problems that are not one of these types. Nevertheless, it will be helpful to know how to do all these types.)

Sect. 1.7: Direct proof, proof by contraposition, proof by contradiction. If and only if proof.

Sect. 1.8: Proof by cases. Existence and uniqueness proofs.

Sect. 2.1: In examples: Which sets are subsets? Drawing and using Venn diagrams. Proving set containment and set equality.

Sect. 2.2: In examples: Applying set operations. Verifying (proving) set operation equalities. Venn diagrams of set operations.

Sect. 2.3: In examples: Is $f : A \rightarrow B$ one-to-one, onto? Prove $f : A \rightarrow B$ is one-to-one, onto? Find range of $f : A \rightarrow B$. Apply composition.

Sect. 2.4: Apply recurrence relation. Show that a given formula is a solution of recurrence relation. Use iteration to guess a solution to a recurrence relation. Apply formulas (Table 2) to calculate sums.

Not on exam. Sect. 1.8: Subsections 1.8.8–1.8.9. Sect. 2.2: Subsections 2.2.3–2.2.5. Sect. 2.3: Subsection 2.3.6.