

Format and topics for exam 1
Math 42

General information. Exam 1 will be a timed test of 75 minutes, covering sections 1.1–1.6 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we’ve studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, but you should definitely spend time memorizing the *statements* of the important results in the text, especially any result with a name (e.g., DeMorgan’s Laws).

Types of questions. In general, there are three types of questions that will appear on exams:

1. Statements of definitions and theorems;
2. Computations; and
3. Problem-solving with explanation.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a named theorem from the book. You will not be asked to recite the proofs of any theorems from the book.

Computations. These will be drawn from computations of the type you’ve done on the problem sets. On a straight computational problem, you do not need to explain your answer, but you must show all your work.

Problem-solving with explanation. Many problems in combinatorics involve the application of theory, e.g., determining if a given graph is bipartite. For these problems, you will be asked to solve the problem, and you will also be asked to justify or explain the validity of your solution.

Definitions. The most important definitions and symbols we have covered are:

1.1	proposition	propositional variable
	truth value (of a proposition)	compound proposition
	logical operator	negation $\neg p$
	truth table	conjunction $p \wedge q$ (p AND q)
	disjunction $p \vee q$ (p OR q)	exclusive or $p \oplus q$ (p XOR q)
	conditional $p \rightarrow q$ (IF p THEN q)	converse
	contrapositive	inverse
	biconditional $p \leftrightarrow q$ (p IF AND ONLY IF q)	Boolean variable
	bit operations	bit string
	length (of a bit string)	
1.3	tautology	contradiction
	logically equivalent	satisfiable
1.4	predicate	propositional function
	bound (variable)	free (variable)
	scope (of a quantifier)	
1.5	domain (of discourse)	universal quantifier $\forall x$
	existential quantifier $\exists x$	bound variable
	free variable	
1.6	(logical) argument	premises
	conclusion	valid (argument)
	argument form	

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to cite them as needed. You should also be prepared to recite named theorems.

Sect. 1.1: Truth tables (definitions) for logical operators. $p \rightarrow q$ is equivalent to contrapositive, but not converse or inverse.

Sect. 1.3: DeMorgan's laws for \wedge and \vee . $p \rightarrow q$ written as or statement $\neg p \vee q$. Distributive and associative laws.

Sect. 1.4: DeMorgan's laws for quantifiers.

Sect. 1.6: Rules of inference: modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism

Types of problems. You should also know how to do the following general types of problems, some of which are straight computations, and some of which require explanation. (Note also that on the actual exam, there may be problems that are not one of these types. Nevertheless, it will be helpful to know how to do all these types.)

Sect. 1.1: Negating propositions. Translating $\neg p$, $p \wedge q$, $p \vee q$, $p \rightarrow q$, etc., to English. Making a truth table for a compound proposition.

Sect. 1.2: Translating English to $\neg p$, $p \wedge q$, $p \vee q$, $p \rightarrow q$, etc.

Sect. 1.3: Showing propositions are logically equivalent: Using truth table, using description of when $p \neq q$ is true, applying laws of symbolic logic (DeMorgan, associativity, distributivity, etc.). Showing a proposition is a tautology and showing a proposition is satisfiable: Same techniques as logically equivalent.

Sect. 1.4: Evaluate truth values of propositional functions. Evaluate truth values of quantifications. Quantifiers written out over various domains. Negations of quantifiers. Translating English into symbolic expressions.

Sect. 1.5: Translating multiple-quantifier propositions into English. Evaluate truth values of propositions with nested quantifiers. Writing mathematical statements using nested quantifiers. Translating English into multiple-quantifier propositions. Negating statements with nested quantifiers.

Sect. 1.6: Applying rules of inference. Using rules of inference to build arguments. Rules of inference for quantified statements. Building arguments with propositions and quantified statements.

Not on exam. Sect. 1.2: Subsections 1.2.3–1.2.4 and 1.2.6. Sect. 1.3: Subsections 1.3.6–1.3.7. Sect. 1.4: 1.4.11–1.4.13.