Math 42 Final Spring 2017

Grading
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Your PRINTED name is:________________________

Your exam will receive a 0 without a signature here: On my honor as a human being, I have neither given nor received any help on this exam that the instructor would not approve of. I understand that I will receive an F for the course if I break this promise:

(SIGN:)____________________

Instructions:

1. You have 120 minutes to complete the exam.
2. You must show all of your work to receive full credit.
3. There are 200 points on the exam. Each problem is worth exactly 25 points.
4. No cell phones, notes, books, calculators, use of internet and computer are allowed at any time.
5. Good luck!

On proof questions, remember that you are graded on the presentation of proving your answer, not just getting the answer.

1 (25 pts.)

1. (10 pts) Write the truth table for the proposition $(p \vee \neg q) \rightarrow p$.

2. (15 pts) You have to invite some people to a party. If you invite Lex, you must invite Lois. If you invite Clark, you must invite Lex. You must invite either Lois or Clark, but not both. You must invite either Lex or Clark or both. What are the possible invitations you can make? Prove your answer. (hint: you probably want to convert the conditions into statement forms)

2 (25 pts.)

1. (10 pts) Find the domain, the codomain, and the range of the following function $f : \mathbb{R} \to \mathbb{R}$:

$$f(x) = x^2 - 4.$$

2. (15 pts) Find the inverse of the following function or explain why it is not invertible: $f : \mathbb{R} \to \mathbb{R}$:

$$f(x) = x^3 + 2.$$

3 (25 pts.)

Hint: for this problem, you can assume at least one irrational number exists; in particular, I'll let you assume $\sqrt{2}$ is irrational.

1. (10 pts) Prove or disprove: if x and y are rational, then $x + y$ is rational.

2. (15 pts) Prove or disprove: if x and y are irrational, then $x + y$ are irrational.

4 (25 pts.)

1. (12 pts) Suppose a is an integer where $a \equiv 2 \pmod{21}$. Find the integer c with $0 \leq c \leq 20$ such that $a^6 = c \pmod{21}$. Show your work.

2. (13 pts) Which positive integers less that 30 are relatively prime to 30?

5 (25 pts.)

1. (10 pts) Let $a_1 = 9$ and $a_n = 3a_{n-1}$ for $n \geq 2$. Find an explicit (closed-form) formula for $\{a_n\}$.

2. (15 pts) Find (prove your answer) the sum

$$5 + 10 + 15 + \cdots + 100.$$

6 (25 pts.)

(a) (10 pts) Hackers are trying to hack your Headbook account! Unluckily for you, they heard from you that in your passwords, each character is either a capital English letter or a number. Furthermore, they know your password is exactly 8 characters long (example password that you would use: "IAMCOOL5"). How many passwords do they need to check to ensure finding your password?

(b) (15 pts) After your embarassing photos were taken by hackers, you swear to improve your security. Now, you also can use lower-case letters in addition to capital letters and numbers that you used before. You also change up the length of your passwords so that they are between 8 and 11 (inclusive, so both 8 and 11 are possible) characters long. How many passwords do hackers need to go through now, knowing this information?

7 (25 pts.)

(a) (10 pts) Let n be an integer. Prove or disprove: if n^2+1 is odd, then n must be even.

(b) (25 pts) Let n be an integer. Prove or disprove: if $2|n$ and $3|n$, then $6|n$. (hint: consider $n \pmod 6$, or use unique factorization.)

8 (25 pts.)

Prove that $3^n \geq 4n$ for all integers n with $n \geq 2$.