Math 42, Spring 2023 **Name:** _____
Exam 2

This test consists of 8 questions on 6 pages, totalling 100 points. You are not allowed to use books, notes, or calculators. Unless otherwise stated, you may take as given anything that has been proven in class, in the homework, or in the reading.

1. (10 points) Let A, B, C be sets. Draw the standard Venn diagram for A, B, and C, and shade in the region(s) of your Venn diagram corresponding to $(A \cup B) - C$. No explanation necessary.

2. (12 points) Let n be an integer. Prove that if n is even, then $5n + 7$ is odd. (Use direct proof.)

3. (12 points) Use a proof by cases to show that 26 is not the cube of a positive integer. (Suggested cases: $1 \leq x \leq 2$ and $x \geq 3$.)

4. (12 points) Let A, B, C be sets. Prove that $(B - A) \cap (C - A) \subseteq (B \cap C) - A$. (Use a set containment proof.)

5. (12 points) Let x be a real number. Recall that a real number is rational exactly when it is equal to $\frac{p}{q}$, where $p, q \in \mathbf{Z}$ and $q \neq 0$.

Prove that x is rational if and only if $\frac{2x}{3}$ is rational, as follows:

- Set up the two "if-then" proofs you need to prove the above "if and only if" statement.
- Finish both if-then proofs.

6. (14 points) Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Write down an example of a proper subset C of $A \times B$ such that C contains at least 3 elements. No explanation necessary.

7. (14 points) For each of the following recurrence relations and initial conditions:

- Find the values of $a_0, \ldots, a_5$; and
- Find the solution to the recurrence relation with the given initial conditions.

(a) $a_n = a_{n-1} + (2n - 1)$; $a_0 = -5$.

(b) $a_n = 2a_{n-1} + 1$; $a_0 = 1$.

8. (14 points) Consider the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 5x^3 - 13$.

(a) Define what it means for f to be one-to-one. (I.e., just recite the definition.)

(b) Define what it means for f to be onto. (I.e., just recite the definition.)

(c) Prove that f is one-to-one.

You do **not** need to prove that f is onto.

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