

Math 42, Spring 2023
Exam 1

Name: _____

This test consists of 8 questions on 6 pages, totalling 100 points. You are not allowed to use books, notes, or calculators. Unless otherwise stated, you may take as given anything that has been proven in class, in the homework, or in the reading.

1. (12 points) Consider the statement “If you give a mouse a cookie, he will ask for a glass of milk.”

- (a) Write out the converse of that statement, and write out the contrapositive of that statement. Clearly indicate which is which.
- (b) Exactly **one** of the statements you just wrote (the converse and the contrapositive) is logically equivalent to the original statement. Which is it? Clearly indicate which one is equivalent to the original, converse or contrapositive. No explanation necessary.

2. (12 points) Construct a truth table for the compound proposition $(p \wedge q) \rightarrow r$. Make sure you include a column for each intermediate step.

3. (12 points) One of the following statements is true in the domain of the real numbers, and the other is false.

$$\exists x \forall y (x + y = 0)$$

$$\forall y \exists x (x + y = 0)$$

Circle/box the statement that is **FALSE** and briefly **explain** how you know that statement is false.

4. (12 points) Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives.

- (a) Someone is knocking at the door and ringing the bell.
- (b) Anyone who wants to ride the rollercoaster must buckle their seat belt.

5. (13 points) Recall that on the isle of knights and knaves, knights always tell the truth, and knaves always lie. You encounter two residents there, A and B, who say:

A: Either I am a knave or B is a knave.

B: (says nothing)

Determine whether A is a knight or a knave and whether B is a knight or a knave. Briefly **justify** your answer. (Note that not only do you have to show that your answer works, but also that no other answer is possible.)

6. (13 points) Let $B(x, y)$ be the statement “ x has defeated y in combat.” Consider the statement “Someone has defeated a member of the Wu-Tang clan in combat.”

- (a) Express that statement using quantifiers, where the domain of x is all people and the domain of y is all members of the Wu-Tang clan.
- (b) Form the negation of that statement so that no negation is to the left of a quantifier.
- (c) Express the negation in simple English.

7. (13 points) Show that the argument form with premises $q \rightarrow s$, $\neg p \vee q$, $\neg r$, and $\neg p \rightarrow r$ and conclusion s is valid. In other words, use the rules of inference to show that the premises $q \rightarrow s$, $\neg p \vee q$, $\neg r$, and $\neg p \rightarrow r$ imply the conclusion s .

Make sure to number and **JUSTIFY** each step of your argument. You do not have to label each step modus ponens, modus tollens, etc., but for any given step, please indicate which previous step(s) (or “premise”/“given”) imply that step, e.g., “by (4) and (6)”.

8. (13 points) Use a chain of logical identities to show that $(p \vee (p \wedge q)) \rightarrow p$ is a tautology. Clearly label each time you use either DeMorgan's laws or distributivity, but otherwise, you do not need to label the reason for each step.

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