Final Exam

Math 42

Fall 2020

NAME:

Instructions:

1. You have 120 minutes to complete the exam.
2. You have 15 minutes to upload the exam into the Gradescope.
3. For each 1 minute of the submission after the due time, you will lose 3 points.
4. You must show all of your work to receive full credit. Be sure to read all directions and provide explanations when requested.
5. There are 100 points on the exam. You should have 9 pages including the cover page.
6. No cell phones, notes, books, use of internet and computer are allowed at any point during the exam (except for the submission time).
7. Good Luck!

1. (a) (5 points) Write the truth table for the proposition $p \to (q \to \neg p)$.

(b) (5 points) Negate the following statements. As usual, push the negation as far into the statements as possible.

$$\forall x \exists y (P(x) \to (P(x) \vee Q(y)))$$

2. A ternary string is a sequence of digits, where each digit is either 0, 1, or 2.

   (a) (4 points) Use the product rule to determine how many ternary strings of length n do not start with 0.

   (b) (4 points) How many ternary strings of length 20 contain exactly five 1's and exactly six 2's?

   (c) (5 points) What is the probability that a random ternary string of length 20 contains **at least** one 2?

   (d) (5 points) What is the probability that a random ternary string of length 20 contains **exactly** three 0s?

3. (a) (5 points) Let A, B, C be subsets of the universal set U. Draw the Venn Diagram for $(A \cup B) \cap C$.

(b) (4 points) Recall that $[a, b] = \{x|\ a \leq x \leq b\}$ and $(a, b] = \{x|\ a < x \leq b\}$. Write $(2, 6) - (3, 7]$ as an interval.

(c) (5 points) Determine if the following function is one-to-one. Explain.

$$f : \mathbb{R}^+ \to \mathbb{R}^+$$
$$f(x) = x^2$$

4. (a) (6 points) Suppose a and b are integers, $a \equiv 1$ (mod 9) and $b \equiv 6$ (mod 9). Find the integer c with $0 \leq c \leq 8$ such that $a - 4b \equiv c$ (mod 9). Show your work.

(b) (6 points) Find $gcd(48, 270)$. Show your work.

5. (a) (4 points) List all elements in the relation $R = \{(a, b)|a, b \in \mathbb{Z}\ and\ ab = 6\}$.

(b) (6 points) Let $R = \{(a, b), (c, d), (d, c), (e, e)\}$ be a relation defined on $\{a, b, c, d, e\}$. Determine if R is reflexive, antisymmetric, or transitive. Explain.

6. (a) (5 points) Find a recurrence relation for the following sequence:

$$1, 1, 1, 3, 5, 9, 17, \ldots$$

(b) (5 points) List the first 5 elements of the sequence defined below.

$$a_1 = 3$$

$$a_n = 2a_{n-1} - 1, n \geq 2$$

7. (a) (7 points) Let n be an integer. Prove that if $5n + 1$ is odd, then n is even.

(b) (7 points) Use induction to prove that $2^n \geq 6n$ for all integers n with $n \geq 5$.

8. (2 points each) Mark each statement true or false. No need for explanation.

   (a) $\{x\} \in \{x\}$.

   (b) If $A \subseteq B \cup C$, then $A \subseteq B$ or $A \subseteq C$.

   (c) $|A \times B| \geq |A|$ for all sets A and B.

   (d) The multiplication of any rational number with an irrational number is irrational.

   (e) In any group of 25 or more people there are at least three of them who were born in the same month.

   (f) Suppose there are 4 different types of ice cream you like. You must eat at least 25 random ice creams to guarantee that you have had at least 6 samples of one type.