Sample Exam 2
Math 32, Fall 2015

1. (12 points) Suppose $f(x, y)$ is a function such that

$$f(3,4) = 2, \qquad f_x(3,4) = 0, \qquad f_y(3,4) = -2,$$
$$f_{xx}(3,4) = 1, \qquad f_{xy}(3,4) = 0, \qquad f_{yy}(3,4) = -3.$$

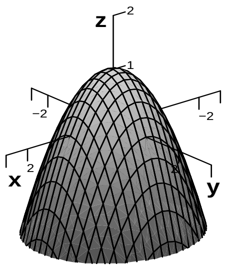
Is $(3, 4)$ a critical point of f? Briefly (1 or 2 sentences) **EXPLAIN** your answer.

2. (18 points)

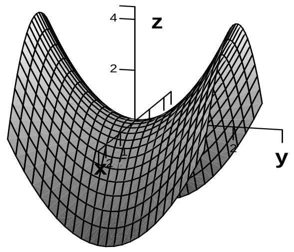
(a) Let $f(x, y) = \ln\left(x^2 + 3y + 1\right)$. Calculate the first partial derivatives of f. No explanation necessary, but show all your work. Do not simplify your final answer.

(b) Let $g(x, y)$ be a function such that $g_x(x, y) = ye^{(x-4)y}$, $g_y(x, y) = (x - 4)e^{(x-4)y}$. Calculate the second partial derivatives of g, including the mixed partials $g_{xy} = g_{yx}$. No explanation necessary, but show all your work. Do not simplify your final answer.

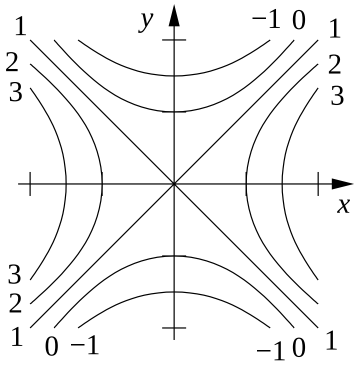
3. (20 points) Consider the following graphs, contour maps, and formulas:



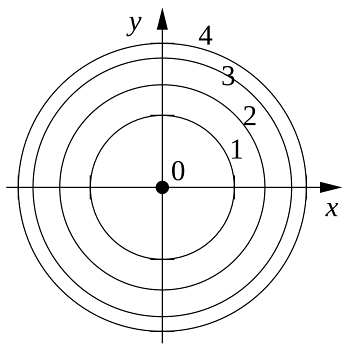
(a)



(b)



(c)



(d)

(1) $z = x^2 + y^2$ (2) $z = 1 + x^2 + y^2$ (3) $z = -x^2 - y^2$ (4) $z = 1 - x^2 - y^2$

(5) $z = x^2 - y^2$ (6) $z = y^2 - x^2$ (7) $z = 1 + x^2 - y^2$ (8) $z = 1 + y^2 - x^2$

For each of the graphs (a) and (b) and the contour maps (c) and (d), find the formula from (1)–(8) that best matches the graph/contour map, and briefly (1 sentence) **EXPLAIN** your answer. In particular, make sure to explain why your answer is better than other similar possibilities.

4. (12 points) Let $f(x,y) = \cos(x^2 - y)$, and let $x = g(t)$ and $y = h(t)$ be differentiable functions such that

$$x(-2) = g(-2) = 4, \qquad y(-2) = h(-2) = 3,$$
$$x'(-2) = g'(-2) = -5, \qquad y'(-2) = h'(-2) = 7.$$

Let $z = F(t) = f(g(t), h(t))$. Use the chain rule to calculate $\frac{dz}{dt} = F'(t)$ at $t = -2$, $x = 4$, $y = 3$. No explanation necessary, but show all your work, and do not simplify your final answer. (In particular, leave expressions like $\cos(271)$ and $\sin(-33)$ as is.)

5. (14 points) Let $g(x,y) = \dfrac{x^3 - 4x}{1 + y^2}$. Find the equation of the tangent plane to $z = g(x,y)$ at $(x,y) = (-1,3)$. No explanation necessary, but show all your work, and do not simplify your final answer.

6. (24 points) A bug is crawling around the xy-plane, looking for warmth, and is currently at the point $(1,3)$. Let $T(x,y)$ be the temperature at the point (x,y) on the plane, in °F, and suppose we know that

$$T(1,3) = 56, \qquad \operatorname{grad} T(1,3) = \nabla T(1,3) = \langle -5, 7 \rangle .$$

(a) Let $\mathbf{u}$ be the unit vector $\mathbf{u} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$. Calculate the directional derivative $D_{\mathbf{u}}T(1,3)$.

(b) If the bug wants to get warmer as quickly as possible, should it crawl north, south, east, west, northeast, southeast, northwest, or southwest? Briefly **explain** your answer. If you are not familiar with north, south, etc., the directions can be summarized as:

NW	N	NE
W	+	E
SW	S	SE

(c) Use the linear approximation to $T(x,y)$ at $(1,3)$ to approximate the temperature at $(1.01, 2.98)$. Show all your work.