Sample Final Exam
Math 32, Fall 2015

1. (12 points) Let $\mathbf{a} = \langle 2, -3, 4 \rangle$ and $\mathbf{b} = \langle 1, 1, 2 \rangle$.

(a) Calculate the dot product $\mathbf{a} \cdot \mathbf{b}$. Show all your work.

(b) Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Without further calculation, determine whether θ is acute ($< \pi/2$), right ($= \pi/2$), or obtuse ($> \pi/2$), and explain your answer in **ONE SENTENCE**.

2. (16 points) Let D be the region in the xy plane with $x \geq 0$ and boundaries $x = 0$, $y = 0$, and $y = 4 - x^2$.

(a) Sketch D.

(b) Express the double integral $\iint_D f(x, y)\, dA$ (where $f(x, y)$ is an unspecified continuous function) as an iterated integral in x and y. **DO NOT EVALUATE THIS INTEGRAL** (i.e., set up the integral, but do not calculate it).

3. (14 points) Consider the parametric curve

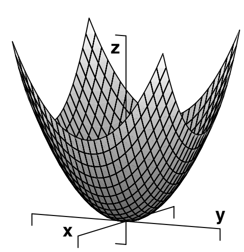
$$x = 1 + 2\cos t, \qquad\qquad y = 2\sin t.$$

Sketch the curve, using arrows to indicate the direction of increasing t.

4. (16 points) Let E be the upper half ($z \geq 0$) of the sphere of radius 7 centered at the origin. Express the triple integral $\iiint_E (x^2 - yz^3)\, dV$ as an interated integral in **spherical coordinates**. **DO NOT EVALUATE THIS INTEGRAL.** Show all your work.

5. (14 points) Let $f(x, y) = \sqrt{x^2 + 3xy}$. Find an equation for the tangent plane to $z = f(x, y)$ at $(x, y) = (1, 3)$. No explanation necessary. **DO NOT SIMPLIFY** your final answer.

6. (14 points) Which of the following equations $z = f(x, y)$ best matches the graph shown below?



(a) $z = x^2 + y^2$ (b) $z = x^2 - y^2$ (c) $z = -x^2 + y^2$ (d) $z = -x^2 - y^2$

Circle the correct equation, and briefly **EXPLAIN** your answer. (In particular, explain why the other equations do not match as well as your choice.)

7. (14 points) Let $f(x,y) = y\sin(x^2y)$.

(a) Calculate f_x and f_y (i.e., calculate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$). Show all your work.

(b) Use part (a) to answer the question: If $(x,y) = (-1,2)$, and we increase x, holding y constant, will $f(x,y)$ increase or decrease? Show all your work, and explain your answer in **ONE SENTENCE**.

8. (16 points) Let $\mathbf{v} = \langle 1,4,0\rangle$ and $\mathbf{w} = \langle -2,1,3\rangle$.

(a) Calculate the cross-product $\mathbf{v}\times\mathbf{w}$.

(b) In **ONE SENTENCE**, describe the geometric relationship between $\mathbf{v}\times\mathbf{w}$ and the vectors $\mathbf{v}$ and $\mathbf{w}$.

(c) Find an equation for the plane that passes through the point $(5,-3,2)$ and is **parallel** to both $\mathbf{v}$ and $\mathbf{w}$. **DO NOT SIMPLIFY** your final answer.

9. (16 points) Let D be the bottom half ($y \le 0$) of the circle $x^2+y^2 = 25$ in the xy-plane. Calculate the double integral

$$\iint_D yx^2\,dA.$$

Show all your work.

10. (16 points) Lorcan is wandering around a strange hilly landscape described by the function

$$z = h(x,y) = 50xy + 200e^{-x^2-y^2},$$

where $h(x,y)$ represents the height above sea level of the point with x and y coordinates (x,y). In other words, Lorcan is walking on top of the graph $z = h(x,y)$.

(a) Calculate the gradient of $h(x,y)$. Show all your work.

(b) Suppose Lorcan is currently at the point $(1, 0.5, h(1,0.5))$. Suppose also that the directions north, south, east, west, etc., correspond to the xy plane in the usual manner:

NW	N	NE
W	+	E
SW	S	SE

I.e., east is the positive x direction, north is the positive y direction, and so on.

If Lorcan wants to go **uphill** as quickly as possible, in which direction should he travel, north, south, east, west, northeast, southeast, northwest, or southwest? Use your results in part (a) to **justify** your answer.

11. (16 points) Let E be the region given by $-2 \le x \le 3$, $1 \le y \le 5$, $0 \le z \le 7$. Calculate the triple integral

$$\iiint_E (x^3y + ze^{2x})\,dV.$$

Show all your work.

12. (18 points) Let $f(x, y)$ be a differentiable function described by the following table.

(x, y)	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
$(1, -4)$	0	2	1	0	1	1
$(3, 5)$	6	0	0	-3	-1	-2
$(-2, 7)$	0	0	0	1	3	2
$(4, 1)$	3	0	0	3	-1	5
$(0, -5)$	-2	-1	0	-1	1	-2

Suppose also that $f_y(x, y) \neq 0$ for all (x, y) not listed in the table.

(a) List all of the critical points of f and briefly **EXPLAIN** why these are the **only** possible critical points of f.

(b) Classify the critical points you found in part (a) as a local maximum, local minimum, or saddle point. Show all your work.

13. (18 points) Cadence is flying in a rocket whose position is described by the space curve

$$\mathbf{r}(t) = \langle 20t, 100 \sin t, 1000 + 100 \sin t - 100 \cos t \rangle .$$

(a) Find Cadence's position, velocity, and acceleration at the time $t = \frac{\pi}{4}$. Show all your work.

(b) Suppose "up" is in the direction of positive z, and "down" is in the direction of negative z, and suppose that the directions north, south, east, west, etc., correspond to the xy plane in the usual manner:

NW	N	NE
W	+	E
SW	S	SE

I.e., east is the positive x direction, north is the positive y direction, and so on.

At time $t = \frac{\pi}{4}$, is Cadence going up or down? In which ground direction is she going, north, south, east, west, northeast, southeast, northwest, or southwest? Use your results in part (a) to **justify** your answer.