Sample final exam
Math 30, Fall 2018

As usual, you should treat this sample exam not as a guide to what will be covered in our exam, but as a guide to what the questions will be like. As always, your best guide to what will be on the exam is the homework.

You will be allowed to use the usual calculators and **ONE** 3×5 notecard. Unless otherwise stated, you must show all your work in a problem to receive full credit.

1. (10 points) Let $y = \dfrac{3t^3 - \sin(2t+1)}{7t^2+3}$. Find $\dfrac{dy}{dt}$. **DO NOT SIMPLIFY** your final answer.

2. (10 points) Starting from the fact that $\sqrt{121} = 11$, use a linear approximation to estimate the value of $\sqrt{121.3}$. Show all your work, and round off you final numerical answer to 4 decimal places.

3. (10 points) Let $h(x) = (3^x - 5x)\sqrt[4]{e^{2x}+7}$. Find $h'(x)$. **DO NOT SIMPLIFY** your final answer.

4. (10 points) Find the most general antiderivative of $f(x) = -7e^x + 3x^{-1} + 4x^{-2}$. Show all your work. **DO NOT SIMPIFY** your final answer.

5. (15 points) Use calculus to find the absolute minimum and absolute maximum values of

$$f(x) = \ln(x^3 - 3x^2 + 13)$$

on the interval $[-1, 4]$. Show all your work, and give your final answers correct to four (4) decimal places.

6. (15 points) Wyatt is designing a hollow cylindrical metal can with volume 1000 cm^3. The material used to make the circular top and bottom of the can costs twice as much as the material used to make the side of the can. What dimensions should Wyatt choose in order to minimize the cost of the can?

Show all your work, round off the numerical part of your final answer to three (3) decimal places, and express your final answer in the form of a **complete sentence**, using the correct units.

7. (15 points) Is it possible that there exists a function f with **ALL** of the following properties?

- $f(3) = 7$;
- $\lim_{x\to 3} f(x)$ does not exist;
- $\lim_{x\to 1} f(x) = -4$;
- f is not continuous at $x = 1$; and
- f is continuous for all $x \neq 1, 3$.

If it **IS** possible to find such an f, sketch the graph of one possible f below. (Your graph need not be to scale, but please make sure that the key features of f can be seen clearly.) If it is **NOT** possible to find such an f, briefly (1–2 sentences) **EXPLAIN** why not.

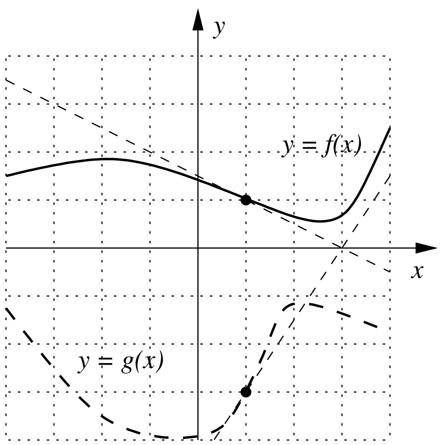
8. (15 points) A brand new crypto-currency, Ittybittycoin, is introduced into the world. A mathematical model shows that t days after Ittybittycoin is introduced, the value of one Ittybittycoin, in dollars, is

$$v(t) = 103 + \frac{\sqrt{t}\cos(t/3)}{12}.$$

At $t = 25$ days, is the value of one Ittybittycoin increasing or decreasing (instantaneously), and at what rate is it increasing or decreasing? Show all your work, round off the numerical part of your final answer to three (3) decimal places, and express your final answer in the form of a **complete sentence**, using the correct units.

9. (15 points) Find the equation of the tangent line to $y = x^2 \ln x + 5x^3$ at $x = 7$. Show all your work, and **DO NOT SIMPLIFY** your final answer.

10. (15 points) Suppose that the graphs of $f(x)$ (heavy solid curve) and $g(x)$ (heavy dashed curve) are as shown below, where each square is 1×1, and the indicated dashed lines are tangent lines to $y = f(x)$ and $y = g(x)$ at the indicated points.



(a) Let $h(x) = (f(x))^5$. Find the value of $h'(1)$. Show all your work.

(b) Let $k(x) = f(x)g(x)$. Find the value of $k'(1)$. Show all your work.

11. (15 points) Olaf is rolling a spherical snowball by packing more snow onto it (while retaining its spherical shape). At one point in time, the radius of the snowball is 3 in, and Olaf is adding snow at a rate of 5 in^3/min. At that instant, is the radius of the snowball increasing or decreasing, and at what rate is it increasing or decreasing?

Show all your work, round off the numerical part of your final answer to three (3) decimal places, and express your final answer in the form of a **complete sentence**, using the correct units.

12. (15 points) Let $f(x) = \dfrac{7x^2 - 3x}{5x^2 + 4x}$. Calculate

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{7x^2 - 3x}{5x^2 + 4x}$$

in one of the following ways:

- Fill in the following table:

x	-0.01	-0.001	0.001	0.01
$f(x)$				

Then guess the value of $\lim_{x\to 0} \frac{7x^2 - 3x}{5x^2 + 4x}$, and **EXPLAIN** in **ONE SENTENCE** how the table leads you to make that guess.

- Use the algebraic limit laws.

You do not need to use more than one method, though you are welcome to do so to check your answer.

13. (20 points) Suppose $g(x)$ is a function such that

$$g'(x) = (x - 3)\sin x.$$

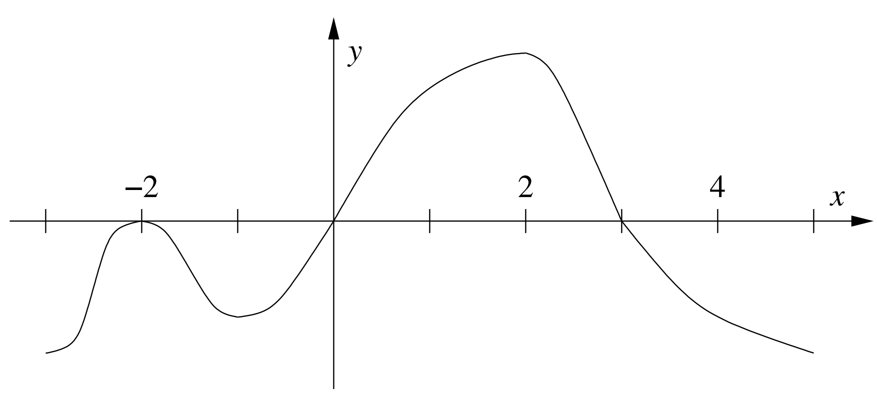
Note that above formula is **THE DERIVATIVE** of g, not g itself.

(a) Find the critical numbers of $g(x)$ for $-1 \le x \le 5$.

(b) Find the values of x $(-1 \le x \le 5)$ for which $g(x)$ is **increasing**, and name the **one** feature of $g'(x)$ that justifies your answer. (I.e., your justification should begin, "$g(x)$ is increasing for/on (blah) because...".)

(c) Find the value(s) of x $(-1 \le x \le 5)$ at which g attains a **relative/local minimum**. Briefly (1 sentence) **JUSTIFY** your answer. (I.e., how do you know that g attains a relative minimum at exactly that/those value(s) of x between -1 and 5?)

14. (20 points) Suppose $f(x)$ is a function whose **DERIVATIVE** $f'(x)$ is graphed below on the domain $-3 \le x \le 5$.



(a) Find the interval(s) of increase and decrease of f for $-3 \le x \le 5$.

(b) Find the x value(s) of the local minima of f for $-3 \le x \le 5$.

(c) Find the x value(s) of the local maxima of f for $-3 \le x \le 5$.

(d) Find the interval(s) of concavity of f for $-3 \le x \le 5$.

(e) Find the x value(s) of the inflection points of f for $-3 \le x \le 5$.

(f) Sketch one possible graph of f for $-3 \le x \le 5$. (There are many different possible answers.) Clearly label all local maxima, local minima, and inflection points on your graph; and make sure that the concavity of $f(x)$ is clear throughout your graph.