

Sample exam 3
Math 30, Fall 2018

Because this semester is more compressed than usual for various reasons (e.g., we are missing a class for Veteran's Day), we will skip section 4.2 completely, and our Exam 3 will cover section 4.3 instead of section 4.2. As a result, this sample is not the Exam 3 I gave last year; instead, I have replaced one problem from last year's exam with a problem from last year's final.

1. (8 points) Let $f(x) = \ln(x^2 + \cos(3x))$. Find $f'(x)$. No explanation necessary. Do not simplify your final answer.

2. (10 points) Suppose that a particle moving along a line has position $r(t) = e^{-t} \cos(2t)$ at time t . Find a formula for the acceleration of that particle at time t . You do not need to simplify your final answer.

3. (12 points) Use implicit differentiation to find an equation of the tangent line to the curve

$$x^3 - xy + y^2 = 11$$

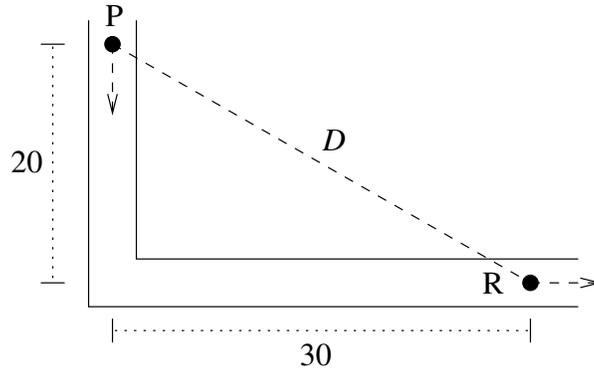
at the point $(2, 3)$. Show all your work, and **DO NOT SIMPLIFY** your final answer (equation of the tangent line).

4. (14 points) The population of Calculus City (the capital city of the country of San Matheo) grows at a rate proportional to its size. At the beginning of the year 2010, the population of Calculus City was 2,173,000, and at the beginning of the year 2015, the population of Calculus City was 2,877,000. How many years before the beginning of 2010 was the population of Calculus City equal to 1,000,000?

Show all your work, round off the numerical part of your final answer to two (2) decimal places, and express your final answer in the form of a **complete sentence**, using the correct units.

5. (14 points) It is a fact that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. Starting from that fact, use a linear approximation to estimate $\sin\left(\frac{\pi}{6} - 0.07\right)$. Show all your work, and round off your final numerical answer to four (4) decimal places.

6. (14 points) Captain Plasma (P) is chasing Reye (R) down an L-shaped hallway, as shown below. At one particular moment, Captain Plasma is 20 m away from the bend of the L, and running towards it at 5 m/s, and Reye is 30 m away from the bend of the L, and running away from it at 4 m/s. At that moment, is the straight-line distance D (see picture below) between Captain Plasma and Reye increasing or decreasing, and at what rate is D increasing or decreasing?



Show all your work, round off the numerical part of your final answer to three (3) decimal places, and express your final answer in the form of a **complete sentence**, using the correct units. Be sure to make it clear whether D is increasing or decreasing.

7. (16 points) Suppose $g(x)$ is a function such that

$$g'(x) = (x - 3) \sin x.$$

Note that above formula is **THE DERIVATIVE** of g , not g itself.

- Find the critical numbers of $g(x)$ for $-1 \leq x \leq 5$.
- Find the values of x ($-1 \leq x \leq 5$) for which $g(x)$ is **increasing**, and name the **one** feature of $g'(x)$ that justifies your answer. (I.e., your justification should begin, “ $g(x)$ is increasing for/on (blah) because...”.)
- Find the value(s) of x ($-1 \leq x \leq 5$) at which g attains a **relative/local minimum**. Briefly (1 sentence) **JUSTIFY** your answer. (I.e., how do you know that g attains a relative minimum at exactly that/those value(s) of x between -1 and 5 ?)

8. (16 points) Use calculus to find the critical numbers and the absolute minimum and absolute maximum values of

$$f(x) = e^{-8x^3 - 3x^2}$$

on the interval $[-2, 5]$. Show all your work, and give your final answers either in exact form or correct to four (4) decimal places.