The Nobel Prize-winning physicist Richard Feynman tells a story in his book *Surely You're Joking, Mr. Feynman* about how, using just pencil and paper, he is able to beat a man with an abacus in a competition of computing cube roots of numbers; specifically, he is able to compute the cube root of 1729.03 faster than the man with the abacus. He goes on to say:

> How did the customer beat the abacus?
>
> The number was 1729.03. I happened to know that a cubic foot contains 1728 cubic inches, so the answer is a tiny bit more than 12. The excess, 1.03 is only one part in nearly 2000, and I had learned in calculus that for small fractions, the cube root's excess is one-third of the number's excess. So all I had to do is find the fraction 1/1728, and multiply by 4 (divide by 3 and multiply by 12). So I was able to pull out a whole lot of digits that way.

1. Feynman used a linear approximation to beat the abacus-user. What function $f(x)$ is being approximated, and near what value $x = a$?
2. Find the linear approximation of $f(x)$ at $x = a$.
3. Find the cube root of 1729.03 using the linear approximation at $x = a$. How far off is the linear approximation from the actual value?
4. Is the linear approximation to the cube root of 1729.03 an underestimate or an overestimate of the actual cube root of 1729.03? Draw a picture to explain.
5. What makes the man's choice of 1729.03 "lucky" for Feynman? What would be some other lucky numbers? What are some numbers that would have been unlucky?
6. The *excess* of a quantity z over/under a is the *relative* change of z from the value $z = a$, i.e., $\dfrac{\Delta z}{a} = \dfrac{(z-a)}{a}$. So what does Feynman mean by saying that "the cube root's excess is one-third of the number's excess"? Figure out the analogous statement for nth roots and justify it with calculus.