

In Section 3.8, we learned the following mathematical models.

- If population  $P(t)$  grows exponentially as a function of time  $t$ , then

$$P(t) = P_0 e^{kt}, \quad (1)$$

where  $P_0$  is the population at time 0 and  $k > 0$  is a positive constant.

- If the mass  $m(t)$  of a radioactive substance decays exponentially as a function of time  $t$ , then

$$m(t) = m_0 e^{kt}, \quad (2)$$

where  $m_0$  is the mass at time 0 and  $k < 0$  is a **negative** constant.

- If  $T(t)$  is the temperature at time  $t$  of an object in a room at surrounding temperature  $T_s$ , and  $y(t) = T(t) - T_s$ , then

$$y(t) = y(0) e^{kt}, \quad (3)$$

where  $y(0) = T(0) - T_s$  and  $k < 0$  is a **negative** constant.

- If you invest  $A_0$  in an account yielding interest rate  $r$ , compounded  $n$  times per year, then  $t$  years later, your balance will be

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}. \quad (4)$$

As  $n \rightarrow \infty$ , we get continuously compounded interest:

$$A(t) = A_0 e^{rt}. \quad (5)$$

1. The half-life of fictionium-17 is 70 years. A team of researchers finds a sample of fictionium-17 that has decayed to 6% of its original mass. How long has the sample been decaying?

Do **NOT** try to solve the problem at first. Instead:

- (a) Identify which of equations (1)–(5) apply to this problem.
- (b) Translate the key parts of the question, word by word, into mathematical statements and questions.

2. If you invest \$17 million in an account that yields 6% interest compounded monthly, how long will it take your investment to reach \$70 million? What is the equivalent annual interest rate (APR)?

Do **NOT** try to solve the problem at first. Instead:

- (a) Identify which of equations (1)–(5) apply to this problem.
- (b) Translate the key parts of the question, word by word, into mathematical statements and questions.

3. An object with temperature  $87^{\circ}\text{F}$  is placed in a room at temperature  $70^{\circ}\text{F}$ , and after one hour, its temperature is  $76^{\circ}\text{F}$ . Find an expression for the temperature of the object at any time  $t$ .

Do **NOT** try to solve the problem at first. Instead:

- (a) Identify which of equations (1)–(5) apply to this problem.
- (b) Translate the key parts of the question, word by word, into mathematical statements and questions.

4. At the beginning of an experiment, a sample contains 17,000 bacteria, and 6 hours later, it contains 70,000 bacteria. How many bacteria will there be after 17 hours?

Do **NOT** try to solve the problem at first. Instead:

- (a) Identify which of equations (1)–(5) apply to this problem.
- (b) Translate the key parts of the question, word by word, into mathematical statements and questions.