

Math 221A, problem set 04
REVISED MON MAR 07
Due: Tue Mar 08
Last revision due: Mon Apr 04

Problems to be turned in: Problem x.y.z of Artin denotes problem y.z in Chapter x.

1. Artin 12.4.5.
2. Artin 12.4.12(b,c).
3. Prove that $x^4 - 12x + 9$ is irreducible over \mathbf{Z} (and therefore, over \mathbf{Q}). Suggestions: Note that Eisenstein's criterion does not apply, so instead, imitate the *proof* of Eisenstein's criterion. You may also find it helpful to consider what happens (mod 9) and to use Lemma 12.4.2.
4. Artin 12.5.5.
5. Artin 12.5.7.
6. Suppose $d = 1 \pmod{4}$ is square-free, $\delta = \sqrt{d}$, and $\eta = (1 + \delta)/2$. Let

$$\mathbf{Z}[\eta] = \{a + b\eta \mid a, b \in \mathbf{Z}\}.$$

Prove that $\mathbf{Z}[\eta]$ is a subring of \mathbf{C} .

7. Artin 13.2.2 (prove your answer).