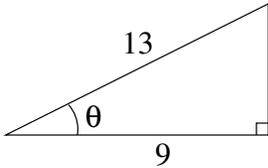


**Sample Exam 4**  
**Math 19, Spring 2013**

- (8 points) Find the exact value (not a decimal approximation) of  $\sin\left(\frac{1}{2}\cos^{-1}\left(-\frac{1}{5}\right)\right)$ . Briefly **JUSTIFY** (in a phrase, sentence, and/or picture) any sign choices (choices of + or -) that you make. Show all your work, and do not simplify your final answer.
- (12 points) Suppose  $x$  is an angle with  $\cos x = \frac{5}{13}$  and  $\sin x = -\frac{12}{13}$ . Find the exact value of  $\cos\left(x + \frac{3\pi}{4}\right)$  (not a decimal approximation). Show all your work, and do not simplify your final answer.
- (8 points) Use the formulas for lowering powers to rewrite  $\sin^2 x + \cos^2(2x)$  as an expression of the form  $a + b\cos(2x) + c\cos(4x)$ , where  $a, b, c$  are constants. (If you can't remember the formulas for lowering powers, start with the formulas for  $\cos(2\theta)$ .)
- (12 points) Find the exact values of the six trigonometric functions of the angle  $\theta$  in the triangle below. No explanation necessary, but show all your work, and do not simplify your answers.



For problems 5–6, **sketch the graph** of the given equation, and **find its period**. Make sure that your graph is aligned with the horizontal and vertical scales, and make sure that you show at least one full period. Finally, make sure that the coordinates of all peaks, valleys, intercepts, and asymptotes are clearly indicated.

- (6 points)  $y = \tan(7x)$ .
- (6 points)  $y = 3\sec(5x)$ .
- (12 points) Verify the trig identity

$$\cot^2 \alpha - \cos^2 \alpha = \cot^2 \alpha \cos^2 \alpha.$$

Show all your work and indicate where, if at all, you use the MOATI (Mother Of All Trig Identities).

- (12 points) In Fairbanks, Alaska, the longest day of the year (June 21) has 23 hours of sunlight, and the shortest day of the year has 3 hours of sunlight.
  - Find a simple harmonic function (i.e., a sin or cos function)  $f(t)$  that models the length of daylight as a function of  $t$ , the number of days after June 21. (In other words, the amount of daylight should be at its peak at  $t = 0$ , and the period of  $f(t)$  should be 365.)

- (b) Sketch the graph of  $f(t)$ . Make sure that your graph is aligned with the horizontal and vertical scales, and make sure that you show at least one full period. Finally, make sure that the coordinates of all peaks, valleys, and vertical midpoints are clearly indicated.

Show all your work.

9. (12 points) Find the exact value (not a decimal approximation) of  $\cot\left(\sin^{-1}\left(-\frac{3}{7}\right)\right)$ . Briefly **JUSTIFY** (in a phrase, sentence, and/or picture) any sign choices (choices of + or -) that you make. Show all your work, and do not simplify your final answer.

10. (12 points) Find the amplitude, period, and phase shift of

$$y = 7 \sin\left(5\left(x - \frac{\pi}{10}\right)\right)$$

and sketch the graph. Make sure that your graph is aligned with the horizontal and vertical scales, and make sure that you show at least one full period. Finally, make sure that the coordinates of all peaks, valleys, and vertical midpoints are clearly indicated. Show all your work.