

**Sample Exam 2**  
**Math 19, Spring 2013**

1. (6 points) Calculate  $\frac{3-2i}{7+i}$ . Show all your work, and write your final answer in the form  $a+bi$ , where  $a$  and  $b$  are real numbers.

2. (12 points) Consider the quadratic function  $f(x) = 2x^2 - 11x + 4$ .

(a) Express  $f(x)$  in standard form.

(b) Find the vertex and the  $x$ - and  $y$ -intercepts of  $f(x)$ .

(c) Sketch the graph of  $f(x)$ , labelling the vertex and intercepts clearly.

Show all your work, and express all answers in **exact** form, i.e., if an answer involves square roots, etc., do not convert the square roots to decimal form.

3. (14 points) Lyrion “(1/3)-man” Cannister of Easteros is buying swords for his army. He calculates that to buy  $x$  swords, it will cost him  $f(x) = 4.5x + 2$  gold pieces.

(a) Find a formula for  $f^{-1}$ . **EXPLAIN** what  $f^{-1}$  represents, in terms of swords and gold pieces, in **ONE SENTENCE**.

(b) Find  $f^{-1}(137)$ . **EXPLAIN** what your answer tells you about swords and gold pieces in **ONE SENTENCE**.

4. (12 points) Let  $r(x) = \frac{3(x-2)(x+7)}{(x-3)(x+5)} = \frac{3x^2 + 15x - 42}{x^2 + 2x - 15}$ .

(a) Find the  $x$ -intercept(s) of the graph of  $r(x)$ , if any.

(b) Find the  $y$ -intercept(s) of the graph of  $r(x)$ , if any.

(c) Find the vertical asymptote(s) of  $r(x)$ , if any.

(d) Find the horizontal asymptotes(s) of  $r(x)$ , if any.

Show all your work, and clearly indicate which answer is which.

5. (6 points) Give an example (i.e., make up an example) of a polynomial function  $y = P(x)$  of degree  $n > 1$  such that as  $x \rightarrow +\infty$ ,  $y \rightarrow -\infty$ , and as  $x \rightarrow -\infty$ ,  $y \rightarrow +\infty$ . No explanation necessary.

6. (12 points) Let  $P(x) = x^4 - 3x^3 + 7x^2 - 10x - 1$  and  $D(x) = x^2 + x - 2$ . Use long division to divide  $P(x)$  by  $D(x)$ , and express the quotient  $P(x)/D(x)$  in the form  $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$ . Show all your work.

7. (6 points) Suppose  $P(x)$  is a polynomial with real coefficients. Is it possible that the zeros of  $P(x)$  are precisely  $x = -3$ ,  $x = 5$ ,  $x = 2 + 5i$ , and no others? **EXPLAIN** your answer in **ONE SENTENCE**.

8. (16 points) Let  $P(x) = x^4 - 4x^3 - x^2 + 16x - 12$ . Find all zeros of  $P(x)$ , and factor  $P(x)$  completely. Show all your work.

9. (16 points) Let

$$f(x) = (-5)(x + 1000)^2(x - 7)^2(x + 999)^3(x - 6) = -5x^8 + \dots$$

- (a) Describe the end behavior of  $f(x)$ . In particular, describe what happens to  $y = f(x)$  as  $x \rightarrow +\infty$  and  $x \rightarrow -\infty$ .
- (b) Find the **sign** of the  $y$ -intercept of  $f(x)$ .
- (c) Find the zeros of  $f(x)$  and the multiplicity of each zero.
- (d) For each zero of  $y = f(x)$ , determine the behavior of the graph  $y = f(x)$  near that zero; in particular, determine if the graph cuts through the  $x$ -axis, bumps the  $x$ -axis, or slides through the  $x$ -axis.
- (e) Sketch the graph of  $y = f(x)$ , making sure that all of the above information is indicated clearly. (It is more important that the key features are visible than for your graph to be drawn to scale.)