

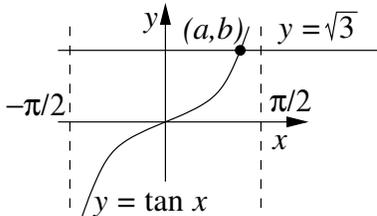
Sample Final Exam
Math 19, Spring 2013
EDITED MAY 8, 2013

1. (6 points) Use the laws of logarithms to expand the expression $\log \left[\frac{(x^5 + 2)^7 (x - 2)^{11}}{(x + 1)^3 \sqrt{x - 5}} \right]$.

Show all your work.

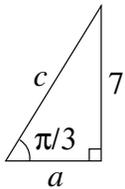
2. (6 points) Is $(-5, 4)$ a point on the graph of the equation $(x + 5)^3 = (2 - y)^2 + 4$? Briefly **JUSTIFY** your answer. Show all your work.

3. (6 points) Find the point (a, b) on the intersection of the graphs of $y = \tan x$ and $y = \sqrt{3}$ such that $-\frac{\pi}{2} \leq a \leq \frac{\pi}{2}$, as sketched in the graph below (not to scale). Show all your work, give **EXACT** values (not decimal approximations), and do not simplify your answer.



4. (6 points) Find the equation of the line with slope -753 that passes through the point $(23, -859)$. No explanation necessary; please do not simplify your answer.

5. (6 points) Find the **EXACT** values (**NOT** decimal approximations) of sides a and c in the right triangle below. (In case this is unclear, the indicated angle is $\frac{\pi}{3}$ radians.) Do not simplify your final answers.



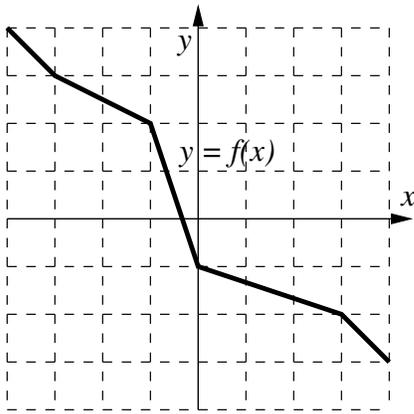
6. (6 points) Consider the rational function $f(x) = \frac{(x + 1)(x + 6)}{5(x - 2)(x - 1)} = \frac{x^2 + 7x + 6}{5x^2 - 15x + 10}$.

(a) Find all vertical asymptote(s) of $f(x)$, if any.

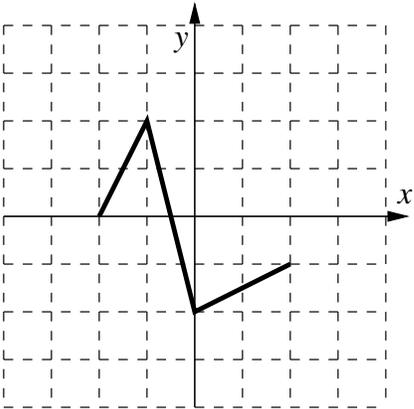
(b) Find all horizontal asymptote(s) of $f(x)$, if any.

No explanation necessary. Write all asymptotes in the form $x = 13$ or $y = -83$, and not just 13 or -83 .

7. (6 points) Suppose $f(x)$ is a one-to-one function whose graph is shown below. (Each square is 1 unit \times 1 unit.) What is the value of $f^{-1}(-3)$? Indicate exactly what information on the graph you are using. (Note that you are given the graph of the *forward* function $y = f(x)$, and your goal is to find one particular value of the *inverse* function f^{-1} .)



8. (6 points) Let $y = f(x)$ be the function whose graph is shown below. (Each square is 1 unit \times 1 unit.) On the same axes, draw the graph of $y = f(x - 1) + 2$. No explanation necessary.



9. (6 points) Solve the inequality $(x - 21)(x - 20)(x + 13) > 0$. Show all your work.
10. (8 points) Graph the function $f(x) = 3 + \left(\frac{1}{2}\right)^x$. Clearly label all x - and y -intercepts of the graph, if any, and list all horizontal and vertical asymptotes, if any. (Write all asymptotes in the form $x = 13$ or $y = -83$, and not just 13 or -83 .)
11. (8 points) For the quadratic function $f(x) = -3x^2 - 4x + 10$:
- Find the vertex of $f(x)$.
 - Find the x - and y -intercepts of $f(x)$. Give **EXACT** values and not decimal approximations.
 - Sketch the graph of $f(x)$, clearly labelling all of the above information.

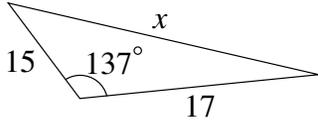
Show all your work.

12. (8 points) Verify the trig identity

$$\sec x - \cos x = \sin x \tan x.$$

Show all your work and indicate where, if at all, you use the MOATI (Mother Of All Trig Identities).

13. (8 points) Find the length of the indicated side x . (The drawing is not to scale.) Show all your work, and round your final numerical answer to the nearest .01.



14. (8 points) Find the average rate of change of $f(x) = 3x^2 - 5x$ between 2 and $2 + h$. Show all your work, and simplify your final answer.

15. (10 points) Find all real zeros of $P(x) = x^3 + x^2 - 5x + 3$, rational and otherwise, and factor $P(x)$ completely. Show all your work.

16. (10 points) Let $\theta = \sin^{-1}\left(-\frac{3}{11}\right)$. Find the **EXACT** values (not decimal approximations) of $\sec \theta$ and $\cot \theta$. Show all your work, and briefly **JUSTIFY** (in a phrase, sentence, and/or picture) any sign choices (choices of $+$ or $-$) that you make. Do not simplify your answers or rationalize denominators.

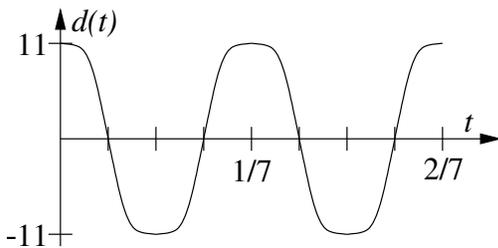
17. (10 points) If current trends continue, the population $P(t)$ (in millions) of California, t years after the beginning of 2012, can be closely modelled by

$$P(t) = 37.3(1.0091^t).$$

Again assuming current trends continue, how many years after the beginning of 2012 will the population of California reach 50 million?

Show all your work, and put your final answer in the form of a complete sentence, using the correct units. Round off your numerical answer to the nearest .1 years.

18. (10 points) Cynthia and Union are experimenting with a mass suspended from a spring. They release the mass at time $t = 0$ and allow it to oscillate, and the graph below gives its displacement $d(t)$ from equilibrium at time t . Express $d(t)$ in the form $d(t) = a \sin(\omega t)$ or $d(t) = a \cos(\omega t)$, whichever is more appropriate. Show all your work.

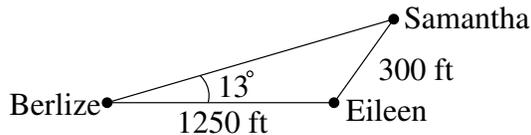


19. (10 points) Graph the equation

$$\frac{x^2}{16} + \frac{y^2}{25} = 1.$$

If this equation is a parabola, label the vertex of the parabola; if this equation is an ellipse, label the vertices and the center of the ellipse; and if this equation is a hyperbola, label the vertices and the center of the hyperbola, and label each asymptote of the hyperbola with its equation. Show all your work.

20. (10 points) Surveyors Berlize, Eileen, and Samantha are taking measurements in a field. As shown below (not to scale), Eileen is 1250 feet from Berlize, Samantha is 300 feet from Eileen (and farther away from Berlize than Eileen is), and the indicated angle is 13° . How far away is Samantha from Berlize? Show all your work, round your final numerical answer to the nearest .1, and write your final answer in the form of a complete sentence, using the correct units.



21. (10 points) For the function $y = 7 \sin(13x)$:

- Find the amplitude.
- Find the period.
- Graph the function. Make sure that your graph is aligned with the horizontal and vertical scales, and make sure that you show at least one full period. Finally, make sure that the coordinates of all peaks, valleys, and vertical midpoints are clearly indicated.

22. (12 points) Suppose θ is in Quadrant III and $\cos \theta = -\frac{5}{17}$. Find the **exact** values (not the decimal approximations) of $\cos 2\theta$ and $\sin 2\theta$. Show all your work, and briefly **JUSTIFY** any sign choices you make. Please do not simplify your final answers.

23. (12 points) Exactly one of the following is the equation of a hyperbola:

$$13x^2 - 7y^2 = 91, \qquad 13x^2 + 7y^2 = 91. \qquad (1)$$

- CIRCLE** the equation that is the equation of a hyperbola.
- Graph the hyperbola, label its vertices, and label each asymptote with its equation. Show all your work.

24. (12 points) Let $f(x) = (x - 1)(x - 2)^3(x + 1000)^2(x + 1001) = x^7 + \dots$

- Find the sign of the y -intercept of $f(x)$.
- Find the zeros of $f(x)$.
- For each zero of $y = f(x)$, determine if the graph bumps the x -axis, cuts the x -axis, or slides through the x -axis at that zero.
- Sketch the graph of $y = f(x)$, making sure that all of the above information is indicated clearly, and that your graph exhibits the proper end behavior. (It is more important that the key features are visible than for your graph to be drawn to scale.)