Everything you ever wanted to know about quadratic functions
(through the Magic Number method)
Math 19

Suppose we have the quadratic function

$$f(x) = ax^2 + bx + c.$$

Here's how to find out everything you would ever want to know about $f(x)$, without having to do the guesswork involved in completing the square. (Or, for that matter, without making the sign errors that are so easy to make when you complete the square.)

1. As the title says, compute the magic number h and $k = f(h)$:

   $$h = -\frac{b}{2a}, \qquad k = f(h).$$

   The point (h, k) then becomes the key to most of what happens afterwards; for example, (h, k) is the vertex of the parabola.

2. Express the quadratic function $f(x)$ in standard form as

   $$f(x) = a(x - h)^2 + k,$$

   where a is the same a from the original formula for $f(x)$. Note that this gives a guesswork-free way to complete the square!

3. The y-intercept of the graph is (as usual) $y = f(0)$, and for the x-intercept(s) of the graph, we solve $f(x) = 0$ using the quadratic formula:

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

   Alternately, you can just set the standard form of $f(x)$ equal to 0 and solve. This is probably easier to do each time than to memorize, but the result comes out to be:

   $$x = h \pm \sqrt{-\frac{k}{a}}.$$

   Note that if $\frac{k}{a} > 0$, then there are no x-intercepts; if $\frac{k}{a} = 0$, then there is one x-intercept; and if $\frac{k}{a} < 0$, then there are two x-intercepts.

4. The maximum or minimum value of $f(x)$ is found at the vertex of the parabola. If $a > 0$, the graph is a smiley face, and $f(x)$ has a minimum of k at $x = h$; if $a < 0$, the graph is a frowny face, and $f(x)$ has a maximum of k at $x = h$.

Once you have all of the above information, when you sketch the graph of $f(x)$, make sure you account for all of it: smiley/frowny face, vertex, y-intercept, x-intercepts.