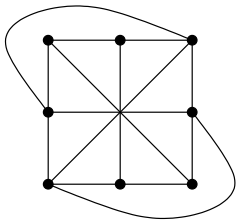
Sample Exam 1
Math 142, Fall 2022

1. (10 points) State Kuratowski's Theorem (the TONCAS theorem for planarity).

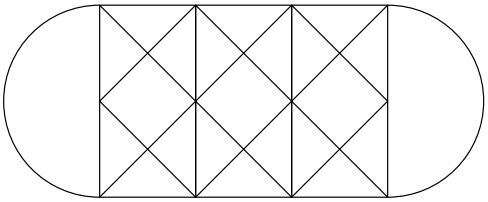
2. (12 points) Anil, Bridget, Carlos, David, and Eun-Mi are friends. David owes money to Anil, Bridget, and Eun-Mi, Carlos owes money to Bridget, and Bridget owes money to Anil and Eun-Mi.

Model this situation with a graph. Make sure that your graph contains all of the above information about who owes money to whom. (You may abbreviate the names as A, B, C, D, and E.)

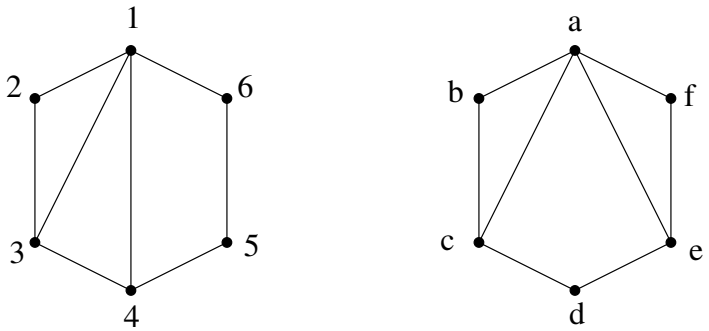
3. (12 points) Determine if the following graph is bipartite. If not, **EXPLAIN** why not, and cite/quote any relevant theorems.



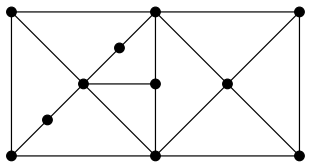
4. (16 points) Can you draw the picture below in one continuous motion, without picking up your pen, and without retracing any lines? Briefly **EXPLAIN** your answer.



5. (18 points) Are the following graphs isomorphic? Briefly **JUSTIFY** your answer.



6. (18 points) Consider the following graph G:



(a) Find a Hamilton (i.e., Hamiltonian) path in G.

(b) Carefully prove that G has no Hamilton circuit. You may explain your proof in words or pictures.

7. (14 points) Let G be a graph with 100 vertices such that the degree of every vertex is at least 5. Explain why it is that if we draw G in the plane, then that drawing will have at least 152 regions.