

Format and topics for exam 1
Math 142

General information. Exam 1 will be a timed test of 50 minutes, covering sections 1.1–1.4 and 2.1–2.2 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets (including problems to be done but not to be turned in) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we’ve studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, but you should definitely spend time memorizing the *statements* of the important theorems in the text, especially any result with a name (e.g., Euler’s Formula).

Types of questions. In general, there are four types of questions that will appear on exams:

1. Statements of definitions and theorems;
2. Computations;
3. Problem-solving with explanation;
4. Proofs.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a named theorem from the book. You will not be asked to recite the proofs of any theorems from the book.

Computations. These will be drawn from computations of the type you’ve done on the problem sets. On a straight computational problem, you do not need to explain your answer, but you must show all your work.

Problem-solving with explanation. Many problems in combinatorics involve the application of theory, e.g., determining if a given graph is bipartite. For these problems, you will be asked to solve the problem, and you will also be asked to justify or explain the validity of your solution.

Proofs. These will resemble the proof questions that have been assigned in the homework, though they will generally be shorter. You should answer in complete sentences, if you have time, but you won’t have to write a lot to answer any given question; to be more precise, you shouldn’t have to write more than a few sentences to answer any given question.

Definitions. The most important definitions and symbols we have covered are:

1.1	graph	vertices
	edges	adjacent
	directed graph	directed edges
	path	circuit
	bipartite	degree (of a vertex)
	independent set	
1.2	isomorphic	isomorphism
	subgraph	complete graph K_n
	complement	in-degree
	out-degree	
1.3	components	length (of a circuit/path)
	2-coloring	
1.4	planar	circle-chord method
	subdivided	complete bipartite graph $K_{r,s}$
	$K_{3,3}$ configuration	K_5 configuration
2.1	multigraph	trail
	cycle	Euler cycle
	Euler trail	
2.2	Hamilton circuit	Hamilton path

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to cite them as needed. You should also be prepared to recite named theorems.

Sect. 1.2: Techniques for determining if graphs are isomorphic (comparing sets of vertex degrees, complements, etc.).

Sect. 1.3: Sum of vertex degrees (Thm. 1), TONCAS theorem for bipartite graphs. Algorithm for detecting bipartite graphs.

Sect. 1.4: Kuratowski's Theorem (TONCAS theorem for planar graphs). Euler's Formula, corollaries to Euler's formula.

Sect. 2.1: TONCAS theorem for Euler cycles and corollary (TONCAS for Euler trails). Algorithm for constructing Euler cycle.

Sect. 2.2: Rules for showing that a graph has no Hamilton circuits.

Types of computations. You should also know how to do the following general types of problems, some of which are straight computations, and some of which require explanation. (Note also that on the actual exam, there may be problems that are not one of these types. Nevertheless, it will be helpful to know how to do all these types.)

Sect. 1.1: Modelling using graphs, modelling using directed graphs.

Sect. 1.2: Determining if two graphs are isomorphic.

Sect. 1.3: Determining if a graph is bipartite, and giving it a bipartite structure (2-coloring), if possible.

Sect. 1.4: Showing a graph is non-planar (bunny-hunting, using corollaries of Euler's Formula).

Sect. 2.1: Constructing an Euler cycle on a graph.

Sect. 2.2: Showing a graph has no Hamilton cycle.

Not on exam. Sect. 1.1: edge covers, interval graphs, vertex bases. Sect. 1.4: dual graph. Sect. 2.2: Results of Dirac, Chvatal, and Grinberg; tournaments, hypercubes, Gray codes.

Good luck.