

Two recurrence relation problems
Math 142

1. (Posed by Kashif Naqvi) The Boston Red Sox and the Chicago Cubs are playing each other in the World Series, which is a best-of-7 series. (In other words, the first team to win 4 games wins the series.) Assume that the two teams are evenly matched; i.e., each team has a 50% chance of winning any particular game. Let $p(m, n)$ be the probability that the Red Sox will win the series, if they have previously won m games and the Cubs have previously won n games. How can we compute $p(m, n)$? What if we have a best-of-3 series, a best-of-5 series, best-of-11, etc.?
2. (Source: <http://perl.plover.com/qotw/>, expert quiz 002) Now suppose that the Red Sox and the Cubs are playing a best-of-5 series. You want to bet \$80 on the Red Sox to win the series, but you can only make bets on individual games. Consider the following instructions:
 - Bet \$30 on each of the first two games.
 - Bet \$20 on the third game if either team has won both of the first two games, and bet \$40 otherwise.
 - Bet \$40 on the fourth game, if there is one.
 - Bet \$80 on the fifth game, if there is one.

In other words:

If the series stands at:	You bet:
0 to 0	\$30
1 to 0	\$30
1 to 1	\$40
2 to 0	\$20
2 to 1	\$40
2 to 2	\$80

(Note that “2 to 0” means that either the Red Sox have won 2 and the Cubs 0, or the other way around.)

It turns out that, if you follow these instructions, no matter what happens, at the end of the series, you will be ahead by exactly \$80 if the Red Sox have won the series, and behind by exactly \$80 if the Cubs have won. Why? Can you find similar instructions for a best-of-7, best-of-11, etc., series?