

**Math 142, problem set 10**  
**Outline due: Thu Dec 01**  
**Final version due: Tue Dec 06**

**Problems to be turned in:**

1. Let the sequence  $a_n$  be given by  $a_n = a_{n-1} + (3n - 2)$ ,  $a_0 = 0$ .
  - (a) Find the generating function of  $a_n$ .
  - (b) Use the generating function of  $a_n$  to find a closed formula for  $a_n$  as a sum of binomial coefficients.
2. Let the sequence  $a_n$  be given by  $a_n = a_{n-1} + n^2$ ,  $a_0 = 0$ .
  - (a) Find the generating function of  $a_n$ .
  - (b) Use the generating function of  $a_n$  to find a closed formula for  $a_n$  as a sum of binomial coefficients.
3. Among a particular group of 123 comic book fans, 60 of them read Marvel comics, 57 of them read DC comics, and 56 of them read Image comics; 30 read both Marvel and DC, 20 read Marvel and Image, and 22 read DC and Image; and 7 read all three brands of comics.
  - (a) How many of these fans read neither Marvel, DC, or Image comics?
  - (b) How many read Image but neither DC nor Marvel?
4. How many integers are there between 1 and 165 (inclusive) that are relatively prime to 165? Note that  $165 = 3 \cdot 5 \cdot 11$ .
5. How many arrangements of the word PARTICULAR either end with a consonant other than R, begin with the letter I, or have the two Rs appearing consecutively?
6. Each of the World Cup soccer teams from England, the United States, Iran, and Wales has 26 (distinct) players. How many ways are there to select a super-team of 26 players from among these four teams, if every country must have at least one player on the super-team? (Again, the 26 players on a given country's team should be treated as distinct from each other.)
7. How many arrangements of the word COUSCOUS are there with no two consecutive letters the same?
8. How many integer solutions are there for  $e_1 + e_2 + e_3 + e_4 = 27$  with each  $e_i \geq 0$ ,  $e_1 \leq 7$ ,  $e_2 \leq 13$ ,  $e_3 \leq 17$ , and  $e_4 \leq 19$ ?
  - (a) Solve this problem using inclusion-exclusion.
  - (b) Solve this problem using generating functions.