

Math 142, problem set 05
Outline due: Wed Oct 05
Final version due: Mon Oct 10

For all of the following problems, explain/justify your answer, and write your final numerical answer as a sum or product of factorials, $\binom{n}{k}$, and so on.

Problems to be turned in:

1. In the card game of bridge, each of four players is initially dealt 13 cards. Remember that a standard deck of cards has 13 cards in each of four suits, spades, hearts, diamonds, and clubs.
 - (a) How many possible 13-card hands are there?
 - (b) What is the probability that a random 13-card hand will contain all four aces?
 - (c) What is the probability that a random 13-card hand will have as even a distribution among suits as possible, i.e., 4 cards of one suit and 3 of each of the others?
 - (d) What is the probability that a random 13-card hand will contain at least 9 spades?
2. There are 13 soccer players from Fremont and 9 from Milpitas who are playing a pickup game of soccer.
 - (a) How many ways are there to form two teams of 11? (Note that when you form two teams, it doesn't matter which is the first team and which is the second team.)
 - (b) How many ways are there to form two teams of 11 so that each team will contain at least one player from Milpitas?
3. How many sequences of 13 Xs and 22 Ys are there with no consecutive Xs?
4. Suppose we have a collection of identical pennies, identical nickels, identical dimes, and identical quarters.
 - (a) How many possible collections of 35 coins are there?
 - (b) How many possible collections of 35 coins are there with at least 4 coins of each type?
5. Suppose we are choosing a set of hair ribbons from among large piles of red, orange, yellow, green, blue, indigo, and violet hair ribbons. How many ways are there to choose 12 hair ribbons:
 - (a) With no other conditions?
 - (b) With at most one violet hair ribbon?
 - (c) With at most one blue, at most one indigo, *and* at most one violet hair ribbon?

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6. How many arrangements of the word CALIFORNIA are there:
- (a) With no other conditions?
 - (b) With the consonants in alphabetical order?
 - (c) With the first consonant occurring before the first A?
7. How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 31$ with:
- (a) All $x_i \geq 0$?
 - (b) All $x_i \geq 1$?
 - (c) All $x_i \geq k$, as a function of $k \leq 5$?