

The big board of balls in boxes
Math 142

	s distinct balls in t distinct boxes	s identical balls in t distinct boxes	s distinct balls in t identical boxes	s identical balls in t identical boxes
≤ 1 ball per box, $t \geq s$	$P(t, s) = \frac{t!}{(t-s)!}$ (Sect. 5.2)	$C(t, s) = \binom{t}{s}$ (Sect. 5.2)	1	1
No limits on balls per box	t^s (Sect. 5.1)	Divider Thm: $\binom{s+(t-1)}{s}$ (Sect. 5.3)	Bell numbers $B(s)$	Partitions, $\leq t$ parts
≥ 1 balls per box	$t! \left\{ \begin{matrix} s \\ t \end{matrix} \right\}$ (Sect. 6.4)	Divider Thm, 1/box already, $\binom{s-1}{s-t}$ (Sect. 5.3)	Stirling numbers (second kind) $\left\{ \begin{matrix} s \\ t \end{matrix} \right\}$ (Sect. 6.4)	Partitions, t parts
n_i balls in box i , $\sum n_i = s$	MISSISSIPPI theorem $P(s; n_1, \dots, n_t)$ (Sect. 5.3)	1		
$\leq n_i$ balls in box i , $\sum n_i = s$	Exponential generating functions	Generating function $\prod (1 + \dots + x^{n_i})$ (Sect. 6.1-6.2)		

Balls in boxes	Arrangement/selection
s distinct balls, t distinct boxes	Arranging s objects from total of t (balls represent locations)
s identical balls, t distinct boxes	Selecting s objects from total of t (balls represent chosen objects)