Format and topics
Final exam, Math 131B

General information. The final will be a little less than twice as long as our in-class exams, with 135 minutes in which to complete it. The final will be **cumulative**; in other words, the final will cover the topics on this sheet and also on the previous three review sheets. However, the exam will somewhat emphasize the material listed here from Chs. 4 and 12, and somewhat de-emphasize the material from Chs. 1–3.

As always, most of the exam will rely on understanding the problem sets (especially PS10–11) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

As usual, **no books, notes, calculators, etc., are allowed.**

Definitions. The most important definitions we have covered are:

4.7	Rapidly decaying	Schwartz space $\mathcal{S}(\mathbb{R})$
12.2	Convolution $f * g$ on $\mathbb{R}$	Dirac kernel on $\mathbb{R}$
12.3	Fourier transform on $\mathcal{S}(\mathbb{R})$	Operator notation for Fourier transform
12.4	Inverse Fourier transform	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 4.7: Poly-Gaussian $p(x)e^{-ax^2+bx}$ $(a > 0)$. Non-Schwartz functions.
Sect. 12.2: Gauss kernel.

Theorems, results, algorithms. The most important theorems, results, algorithms, and axioms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 4.7: $\mathcal{S}(\mathbb{R})$ closed under $\frac{d}{dx}$, $+$, multiplication, multiplication by polynomial (Thm. 4.7.2). $f \in \mathcal{S}(\mathbb{R})$ is uniformly continuous (Cor. 4.7.3). Poly-Gaussian in $\mathcal{S}(\mathbb{R})$ (Thm. 4.7.4).
Sect. 4.8: Integration on $\mathbb{R}$: Parts (Thm. 4.8.7), differentiating under the integral sign (Thm. 4.8.8), Fubini's Theorem (Thm. 4.8.11). $\int_{-\infty}^{\infty} e^{-\pi x^2}\,dx = 1$ (Thm. 4.8.6).
Sect. 12.1: Overview: Inversion Theorem (Thm. 12.1.4).
Sect. 12.2: Properties of convolution (Thm. 12.2.3). Limit properties of $f * K_t$ when $K_t(x)$ is a Dirac kernel (Thm. 12.2.5) Gauss kernel is a Dirac kernel (Thm. 12.2.9).
Sect. 12.3: Fourier transforms of transformations of $f \in \mathcal{S}(\mathbb{R})$ (Thm. 12.3.3). Fourier transform preserves $\mathcal{S}(\mathbb{R})$ (Cor. 12.3.5). Transform of $f * g$ (Thm. 12.3.6). Pass the Hat (Thm. 12.3.7). Gauss kernel is its own double Fourier transform (Thm. 12.3.8).
Sect. 12.4: Inversion Theorem in $\mathcal{S}(\mathbb{R})$ (Thm. 12.4.2). Isomorphism Theorem in $\mathcal{S}(\mathbb{R})$ (Thm. 12.4.3).

On exam, but given in table form. You will be given tables of selected examples of integration by parts and special values of $e_n(x)$, just as you were on exam 2.

Not on exam. Everything from Sections 4.7 and 4.8 except what is listed above.

Good luck.