Format and topics
Exam 2, Math 131B

General information. Exam 2 will be a timed test of 75 minutes, covering Sects. 4.1–4.6, 5.1–5.3, 6.1–6.2, 6.4, and 7.1 of the text. The exam will be cumulative only to the extent that these sections rely on previous material; for example, you should still know what the supremum of a subset of $\mathbb{R}$ is. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to define the supremum of a subset of $\mathbb{R}$.

No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets (PS04–06) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Definitions. The most important definitions we have covered are:

4.1	series	sequence of partial sums
	convergent (resp. divergent) series	converge absolutely
	two-sided series	convergence (of two-sided series)
	converge synchronously	
4.2	pointwise convergence	series of functions
4.3	converge uniformly	uniformly Cauchy
4.4	power series	coefficients
	radius of convergence	
4.5	e^x	$\cos x$
	$\sin x$	π
4.6	e_n	
5.2	function space	zero function/vector
	subspace	vector
	$C^0(X)$	$C^r(X)$
	$C^\infty(X)$	$C^r_x(X)$, $C^r_y(X)$
	S^1 domain	L^∞ metric on $C^0(X)$
	linearly independent	span
5.3	dot product in $\mathbb{C}^n$	orthogonal (in $\mathbb{R}^n$)
	orthonormal (in $\mathbb{R}^n$)	$\ell^2(\mathbb{N})$, $\ell^2(\mathbb{Z})$
6.1	trigonometric polynomial of degree N	Laurent polynomial
	Nth Fourier polynomial	$\hat{f}(n)$
6.2	Fourier series	$\sum_{n\in\mathbb{Z}} f_n(x)$
7.1	inner product	inner product space
	inner product norm	orthogonal vectors
	projection of f onto g	$\mathrm{proj}_g(f)$

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 4.1: Geometric series (also two-sided), p-series (also two-sided), alternating p-series.
Sect. 4.2: The six NO's: NO to QB (Exmp. 4.2.2); NO to QC, QD1 (Exmp. 4.2.3); NO to QD2 (Exmp. 4.2.4); NO to QI1 (Exmp. 4.2.5); NO to QI2 (Exmp. 4.2.6).
Sect. 4.3: Still NO: QD1 and QD2 (Exmp. 4.3.15).
Sect. 4.5: e^x and its properties; e^{ix} and its properties; $\cos x$, $\sin x$ and their properties.
Sect. 5.1: Clocks compared using different metrics.

Sect. 6.2: Waves: Square, sawtooth, triangle, x^2, x^3.
Sect. 7.1: Inner product spaces: $\mathbb{C}^n$ with dot product, $C^0(X)$ with L^2 inner product, $\ell^2(X)$ ($X = \mathbb{N}$ or $\mathbb{Z}$).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 4.1: Cauchy Criterion for series (Cor. 4.1.2), Comparison test (Cor. 4.1.4), Absolute convergence (Cor. 4.1.5), ratio test (Thm. 4.1.14).
Sect. 4.3: Basics of uniform convergence (Thm. 4.3.2, Lem. 4.3.3). Uniform convergence iff uniformly Cauchy (Thm. 4.3.5). L^∞ criterion for uniform convergence (Lem. 4.3.6). Weierstrass M-test (Thm. 4.3.7, Cor. 4.3.8). YES answers: QB (Thm. 4.3.11) QC (Thm. 4.3.12), QI1 (Thm. 4.3.13), QI2 (Thm. 4.3.14), QD1 and QD2 (Thm. 4.3.16).
Sect. 4.4: Radius of convergence and term-by-term differentiation (Thm. 4.4.2).
Sect. 4.5: Properties of e^x (Thms. 4.5.2–4.5.4). Properties of e^{ix} and trig functions (Thms. 4.5.7 and 4.5.11).
Sect. 4.6: $\{e_n\}$ are orthonormal in L^2 inner product.
Sect. 5.2: L^∞ is a metric on $C^0(X)$; L^∞ is complete on $C^0(X)$.
Sect. 5.3: L^2 inner product well-defined on $\ell^2(X)$.
Sect. 6.1: Inner product of trigonometric polynomial with e_n ($n = 0$, $n \neq 0$).
Sect. 6.4: Fourier coefficients of derivative. Differentiability vs. rate of decay of $\hat{f}(n)$ (Thm. 6.4.2, Cor. 6.4.3). Fourier series of $f \in C^2(S^1)$ converges to function with same Fourier coefficients (Thm. 6.4.4).
Sect. 7.1: Basic properties of inner products (Thm. 7.1.3). Pythagorean theorem. Properties of projections (Thm. 7.1.11). Cauchy-Schwarz and triangle inequality in an inner product space (Thm. 7.1.12). $\ell^2(X)$ ($X = \mathbb{N}$ or $\mathbb{Z}$) is an inner product space (Thms. 7.1.13 and 7.1.14).

Given on exam. Table of indefinite integrals and facts about e_n from Sect. 4.6 ((4.6.4)–(4.6.7), (4.6.9)–(4.6.13)).

Not on exam. Sect. 4.3: Real-domain version of uniform convergence results (Thm. 4.3.14). Sect. 4.6: Solutions to linear differential equations.

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the proof notes. You do not need to memorize information from the proof notes, but you do need to be able to apply it.

Good luck.