Format and topics
Exam 1, Math 131B

General information. Exam 1 will be a timed test of 75 minutes, covering Ch. 1 and Sects. 2.1–2.5 and 3.1–3.4 of the text. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets (PS01–03) and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. There are four types of questions that may appear on exams in this class, namely:

1. Computations;
2. Statements of definitions and theorems;
3. Proofs;
4. True/false with justification.

Computations. These will be drawn from computations of the type that you have been assigned on the problem sets. You do not need to explain your answer on a computational problem, but show all your work.

Statements of definitions and theorems. In these questions, you will be asked to recite a definition or the statement of a theorem from the book. You will not be asked to recite the proofs of any theorems from the book, though you may be asked to prove book theorems that you might have been asked to prove on problem sets.

Proofs. These will resemble some of the shorter problems from your homework. You may take as given anything that has been proven in class, in the homework, or in the reading. Partial credit may be given on proof questions, so keep trying if you get stuck (and you've finished everything else). If all else fails, at least try to write down the definitions of the objects involved.

True/false with justification. This type of question may be less familiar. You are given a statement, such as:

- For $a, b \in \mathbb{R}$, if $a \geq b$, then $-a \geq -b$.

If the statement is true, all you have to do is write "True". (However, see below.) If the statement is false (like the one above), not only do you have to write "False", but you must also give a reason why the statement is false. Your reason might be a very specific counterexample:

> False. We have $3 \geq 2$, but $-3 < -2$, which means that $-3 \not\geq -2$.

Your reason might also be a more general principle:

> False. For any $a, b \in \mathbb{R}$, if $a > b$, then $-a < -b$, which means that $-a \not\geq -b$.

Either way, your answer should be as specific as possible to ensure full credit.

Depending on the problem, some partial credit may be given if you write "False" but provide no justification, or if you write "False" but provide insufficient or incorrect justification. Partial credit may also be given if you write "True" for a false statement, but provide some partially reasonable justification. (In other words, if you have time, it can't hurt to justify "True" answers.)

If I can't tell whether you wrote "True" or "False", you will receive no credit. In particular, please do not just write "T" or "F", as you may not receive any credit.

Definitions. The most important definitions we have covered are:

2.1	upper bound	sup
	lower bound	inf
	field	ordered field
	(order) completeness	real numbers
2.2	complex numbers	(complex) conjugate
	norm	absolute value
	real part	imaginary part
2.3	metric	metric space
2.4	sequence	subsequence
	limit (of a sequence)	converges, convergent
	diverges, divergent	bounded (sequence)
	open disc $\mathcal{N}_r(z)$	closed disc $\overline{\mathcal{N}_r(z)}$
	complement	open subset of $\mathbb{C}$
	closed subset of $\mathbb{C}$	limit (in a metric space)
	converge (in a metric space)	dense subset of a metric space
2.5	Cauchy sequence	Cauchy completeness
3.1	sequential continuity	ϵ-δ continuity
	continuous on X	metric continuity
	uniform continuity	bounded function
	limit point	sequential limit (of a function)
	ϵ-δ limit (of a function)	piecewise continuous
	piecewise property	intervals of continuity
3.2	differentiable	derivative $f'(a)$
	differentiable on X	path
	path-connected	
3.3	partition	ith subinterval
	refinement	common refinement
	nth standard partition	$m(v;P,i)$, $M(v;P,i)$
	upper (resp. lower) Riemann sum	$U(v;P)$, $L(v;P)$
	upper (resp. lower) Riemann integral	$\overline{\int_a^b} v(x)\,dx$, $\underline{\int_a^b} v(x)\,dx$
	integrable	definite integral
	$\int_a^b v(x)\,dx$	integral, integrable (complex-valued)
	$\mu(f;P,i)$	$E(f;P)$

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 1.1: Motivating example of trigonometric series for x.
Sect. 2.3: Metrics on $\mathbb{R}$ and $\mathbb{C}$.
Sect. 2.4: Example of definition of subsequence; examples of closed subspaces of $\mathbb{C}$ (Thm. 2.4.9).
Sect. 2.5: Example showing $\mathbb{Q}$ is not complete (Exmp. 2.5.3).
Sect. 3.1: Examples of continuous functions: polynomials, $f(x) = d(x, b)$, $|z|$.
Sect. 3.2: Mean Value Theorem fails for complex-valued functions (Exmp. 3.2.11).
Sect. 3.3: Standard partition with n subintervals; continuous functions and standard partition into n pieces. Integral of a constant function.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 2.1: Properties of ordered fields (Thm. 2.1.3); consequences of order completeness (Thm. 2.1.4); Arbitrarily Close Criterion, Sup Inequality Lemma.

Sect. 2.2: Basic properties of complex numbers (Thm. 2.2.3).

Sect. 2.3: Cauchy-Schwarz and triangle for metric on $\mathbb{C}$ (Thm. 2.3.4). Standard metric on $\mathbb{C}$ is, in fact, a metric.

Sect. 2.4: Boundedness properties and limit laws (Thms. 2.4.5–2.4.6). Examples of closed subspaces of $\mathbb{C}$ (Thm. 2.4.10). Limits and real and imaginary parts (Thm. 2.4.12). Arbitrarily Close Criterion (Thm. 2.4.13). Convergence of monotone sequences (Thm. 2.4.14). Metric squeeze lemma (Lem. 2.4.16). Equivalent definitions of dense (Thm. 2.4.17).

Sect. 2.5: Convergent implies Cauchy (Thm. 2.5.2). Cauchy implies bounded (Lem. 2.5.5). Bolzano-Weierstrass in $\mathbb{R}$ (Thm. 2.5.6). $\mathbb{R}$ is Cauchy complete (Thm. 2.5.7), $\mathbb{C}$ is Cauchy complete (Cor. 2.5.8). Bolazno-Weierstrass in $\mathbb{C}$ (Thm. 2.5.9).

Sect. 3.1: Equivalence of sequential and ϵ-δ continuity (Thm. 3.1.4). Continuity laws (Thms. 3.1.5–3.1.7) Uniform continuity on closed and bounded subsets of $\mathbb{C}$ (Thm. 3.1.12); Extreme Value Theorems (Thm. 3.1.15, Cor. 3.1.16); Intermediate Value Theorem (Thm. 3.1.17). Equivalence of sequential and ϵ-δ limits (Thm. 3.1.20). Limit laws (Thm. 3.1.21–3.1.22); Squeeze Lemma (Lem. 3.1.23).

Sect. 3.2: Real-imaginary differentiability (Cor. 3.2.4). Local linearity (Lem. 3.2.5, Cor. 3.2.6). Differentiation implies continuity (Cor. 3.2.7). Differentiation laws (Thms. 3.2.8–3.2.9). Mean Value Theorem (Thm. 3.2.10), (Complex) Zero Derviative Theorem (Cor. 3.2.13).

Sect. 3.3: Finer partitions are better (Lem. 3.3.8). Lower integral is $\leq$ upper integral (Thm. 3.3.9). Sequential criteria for real integrability (Lem. 3.3.10) and complex integrability (Lem. 3.3.12).

Sect. 3.4: Integration laws (Thm. 3.4.2, Cor. 3.4.3). Additivity of domain (Thm. 3.4.4). Continuity implies integrability (Thm. 3.4.5, Cor. 3.4.6). Combination theorems for integrable functions (Thm. 3.4.8). Nonzero continuous functions have nonzero integrals (Lem. 3.4.9).

Not on exam. Sect. 2.1: Axioms for the real numbers; groups and rings. Sect. 2.3: Lem. 2.3.7. Sect. 3.1: Thm. 3.1.24. Sect. 3.2: Cor. 3.2.14, Thm. 3.2.15. Sect. 3.4: Lem. 3.4.7.

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the proof notes. You do not need to memorize information from the proof notes, but you do need to be able to apply it.

Good luck.