Math 131B, problem set 03
Outline due: Mon Sep 16
Complete version due: Wed Sep 18
Last revision due: Tue Oct 22

1. 3.3.2.

2. 3.3.3.

3. Let $v : [0,1] \to \mathbf{R}$ be defined by

$$v(x) = \begin{cases} \dfrac{1}{q} & \text{for } x = \dfrac{p}{q} \in \mathbf{Q} \text{ in least terms,} \\ 0 & \text{for } x \notin \mathbf{Q}. \end{cases}$$

   ("In least terms" means that q is chosen to be the smallest positive integer with $x = p/q$. In particular, $v(0) = 1$.)

   (a) List all values of $x \in [0,1]$ such that $v(x) > 1/4$. (There are five such values.)

   (b) Find a partition $P = \{x_0, \ldots, x_9\}$ (i.e., with 9 subintervals) such that $U(v;P) < .26$, and briefly explain how you can be sure that $U(v;P) < .26$. (It may help to draw a picture.)

   (c) Prove that v is integrable on $[0,1]$, and calculate (with proof) the integral $\displaystyle\int_0^1 v(x)\,dx$.

4. 3.4.3.

5. 3.4.4.

6. 3.4.6.