Outline for PS01
Math 131B

Definitions. The definitions you had to copy were: (2.1) upper bound, sup, lower bound, inf, additive abelian group, commutative ring, field, ordered field, (order) completeness, real numbers.

(2.2) complex numbers, (complex) conjugate, norm, absolute value, real part, imaginary part,

(2.3) metric, metric space.

(2.4) sequence, subsequence, limit (of a sequence), converges, convergent, diverges, divergent, bounded (sequence), open disc $\mathcal{N}_r(z)$, closed disc $\overline{\mathcal{N}_r(z)}$, complement, open subset of $\mathbf{C}$, closed subset of $\mathbf{C}$, limit (in a metric space), converge (in a metric space), dense subset of a metric space.

Problem plans.

1. **A**. $x = p/q$, $p, q \in \mathbf{Z}$, $q \neq 0$.
(stuff)
C. There exist infinitely many $n \in \mathbf{Z}$ such that $\cos(2\pi n x) = 1$.

2. **A**. $S, T \subseteq \mathbf{R}$, S, T nonempty and bounded.
A. For every $s \in S$ and $t \in T$, $s \leq t$.

(a) **A**. $t \in T$
(stuff)
C. $t \geq \sup S$.

(b) (stuff)
C. $\sup S \leq \inf T$.

3. (2.2.3) By contradiction:
A. There is a definition of $\leq$ on $\mathbf{C}$.
(stuff)
C. Contradiction.

2.4.1, 2.4.10.

4. (2.3.1) **A**. $z, w \in \mathbf{C}$
(stuff)
C. $\Re(z\overline{w}) \leq |z|\,|w|$

5. (2.3.5) **A**. X a metric space, $a, b, x \in X$.
(stuff)
C. $d(a,b) - d(b,x) \leq d(a,x) \leq d(a,b) + d(b,x)$.

6. (2.4.1) **A**. $\lim_{n\to\infty} a_n = L \neq 0$.
(stuff)
C. There exists K such that if $n > K$, then $|a_n| \geq |L|/2$.

7. (2.4.10) **A**. x_n sequence in a metric space X, $L \in X$.
d_n sequence in $\mathbf{R}$ such that $\lim_{n\to\infty} d_n = 0$, $d(x_n, L) < d_n$ for all n.
(stuff)
C. $\lim_{n\to\infty} x_n = L$.