

How to do a problem set outline Math 128A

As requested, this is just meant to give a better idea of what a problem set outline is.

Definitions. Here you should list the definitions from Sections 2.1–2.4. (Skip Defn. 2.1.2 except for the paragraph that begins “To give a bit more detail about (OC)...,” which has a lot of very important definitions having to do with bounds.)

Problem plans. Here is the problem plan for 2.1.5, where you have to prove:

Suppose S is a nonempty subset of \mathbb{R} , and suppose u is an upper bound for S . Then the following are equivalent:

1. For every $\epsilon > 0$, there exists some $s \in S$ such that $u - s < \epsilon$.
2. $u = \sup S$.

“The following are equivalent” means that you have to prove two statements: “If 1, then 2,” and “If 2, then 1.” So your proof will have one global assumption, followed by two if-then structures:

A. $S \subseteq \mathbb{R}$, $S \neq \emptyset$, u an upper bound for S .

Part 1:

A. For every $\epsilon > 0$, there exists some $s \in S$ such that $u - s < \epsilon$.

(stuff)

C. $u = \sup S$.

Part 2:

A. $u = \sup S$.

(stuff)

C. For every $\epsilon > 0$, there exists some $s \in S$ such that $u - s < \epsilon$.

A. means “Assume:” and **C.** means “Conclude:”. Anyway, that’s good enough, though Part 2 can be developed further because a “for every” conclusion is really an “if-then” conclusion, which can be unwound further:

Part 2:

A. $u = \sup S$.

A. $\epsilon > 0$.

(stuff)

C. There exists some $s \in S$ such that $u - s < \epsilon$.

C. For every $\epsilon > 0$, there exists some $s \in S$ such that $u - s < \epsilon$.

For more about the idea of proving if-then statements using this assume/conclude method, see my handout about methods of proof.