

**The most important  $\epsilon$ -ish definitions**  
**Math 131A and 131B**

|  | Applies to:   | Definition   |
|--|---|--|
| $\sup S = U$                                       | Nonempty $S \subseteq \mathbf{R}$                     | 1. $U$ is an upper bound for $S$ (i.e., for all $x \in S$ , $x \leq U$ ); and<br>2. If $v$ is an upper bound for $S$ , then $U \leq v$ .             |
| $\lim_{n \rightarrow \infty} a_n = L$              | Sequence $a_n$  | For every $\epsilon > 0$ , there exists $N(\epsilon)$ such that if $n > N(\epsilon)$ , then $d(a_n, L) < \epsilon$ .                                 |
| $f$ continuous at $a$<br>( $\epsilon$ - $\delta$ ) | Function<br>$f : X \rightarrow \mathbf{C}$            | For every $\epsilon > 0$ , there exists $\delta(\epsilon)$ such that if $d(x, a) < \delta(\epsilon)$ , then $d(f(x), f(a)) < \epsilon$ .             |
| $f$ continuous at $a$<br>(sequential)              | Function<br>$f : X \rightarrow \mathbf{C}$            | For every sequence $x_n$ in $X$ , if $\lim_{n \rightarrow \infty} x_n = a$ , then $\lim_{n \rightarrow \infty} f(x_n) = f(a)$ .                      |
| $f_n$ converges to $f$<br>pointwise                | Sequence $f_n$<br>$f_n, f : X \rightarrow \mathbf{C}$ | For every $x \in X$ and every $\epsilon > 0$ , there exists $N(\epsilon, x)$ such that if $n > N(\epsilon, x)$ , then $d(f_n(x), f(x)) < \epsilon$ . |
| $f_n$ converges to $f$<br>uniformly                | Sequence $f_n$<br>$f_n, f : X \rightarrow \mathbf{C}$ | For every $\epsilon > 0$ , there exists $N(\epsilon)$ such that for every $x \in X$ , if $n > N(\epsilon)$ , then $d(f_n(x), f(x)) < \epsilon$ .     |

Note: For each of the  $d(a, b)$  occurring above, if  $a, b \in \mathbf{C}$ , then  $d(a, b) = |a - b|$ ; otherwise,  $d(a, b)$  depends on the metric chosen for the space in which both  $a$  and  $b$  lie.