QD2: If f_n converges to f **uniformly**, f_n and f all differentiable, is lim f '_n = f '?
STILL NO: Take g(x)=0 and:

```
> g_n := x^(n+1)/(n+1);
```

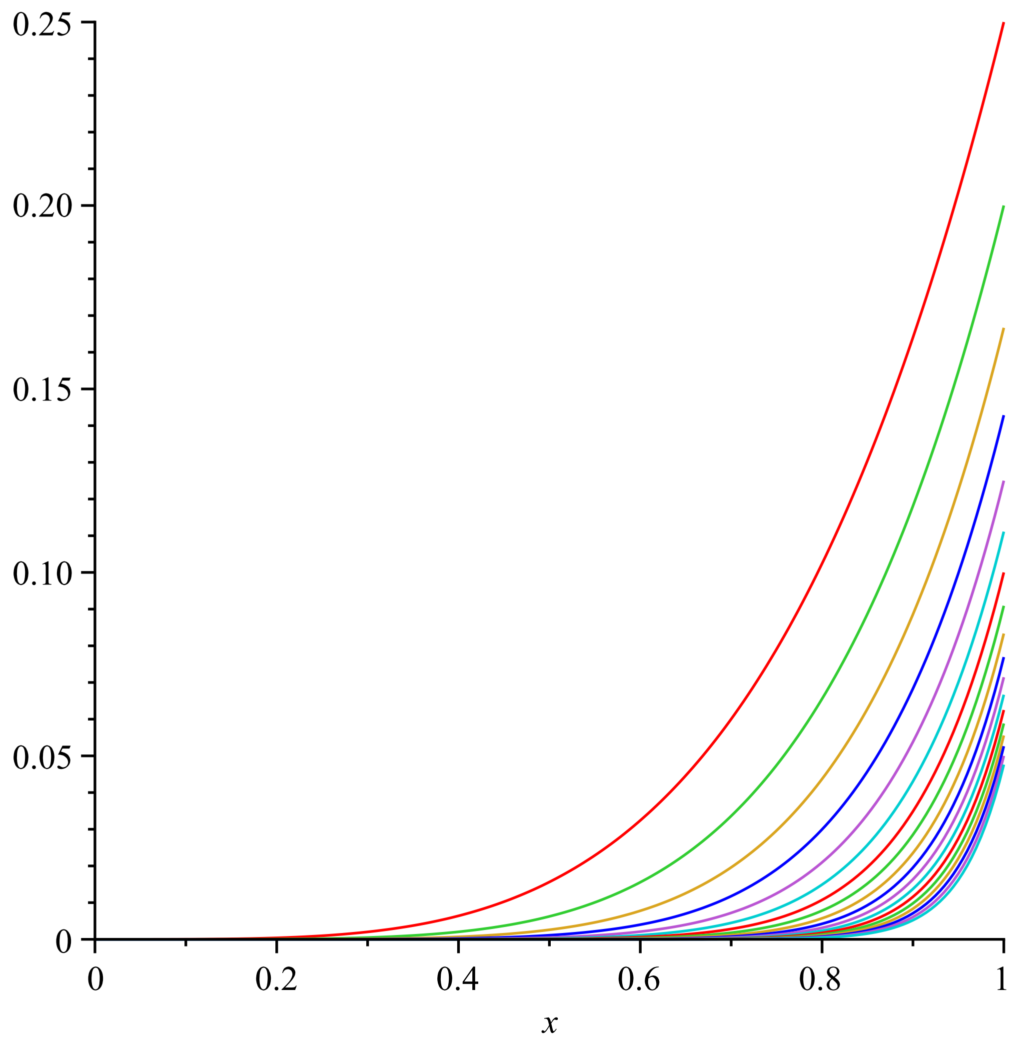
$$g\_n := \frac{x^{n+1}}{n+1} \quad \textbf{(1)}$$

```
> simplify(diff(g_n,x),assume=positive);
```

$$x^n \quad \textbf{(2)}$$

When x=1, lim(x^n)=1. But lim (g_n(x)) =0 for all x in [0,1], and in fact, convergence of g_n is uniform on [0,1].

```
> plot([seq(x^(n+1)/(n+1),n=3..20)],x=0..1);
```



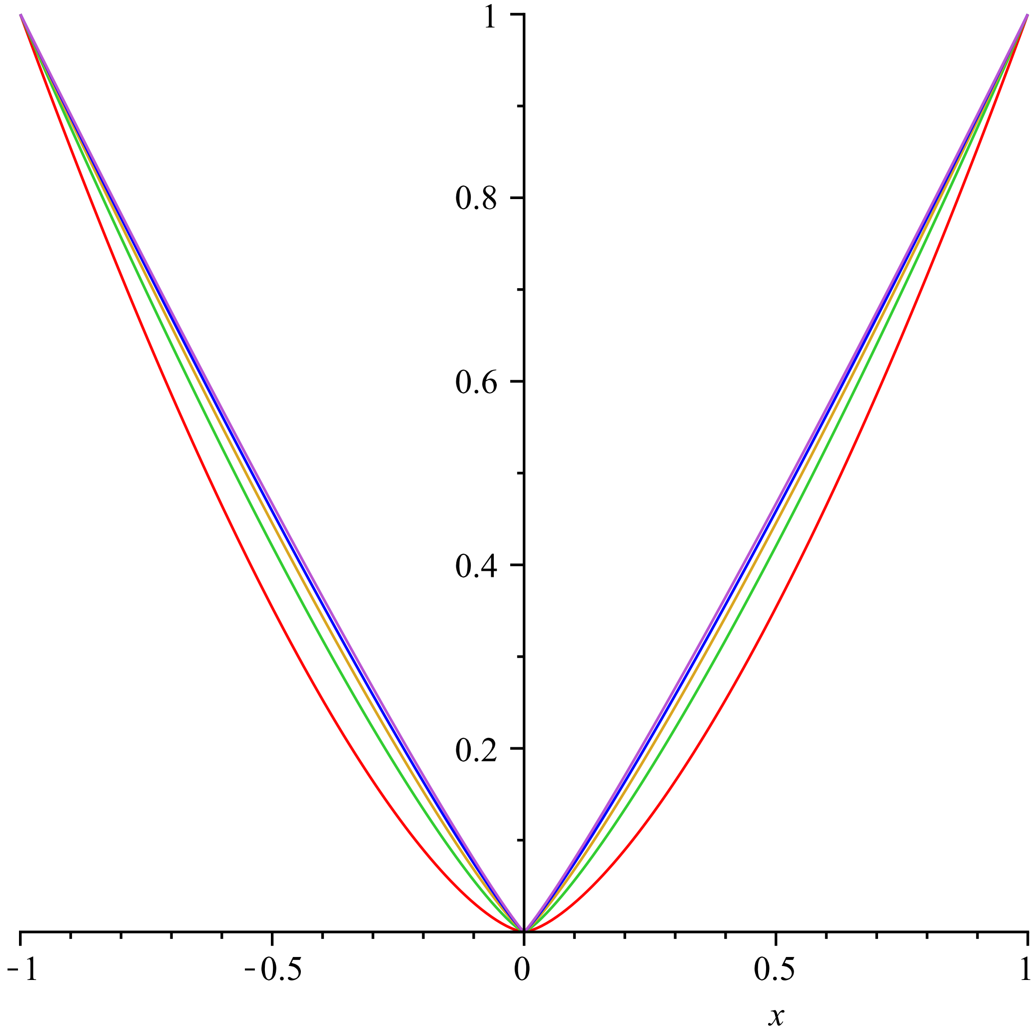
QD1: If f_n converges to f **uniformly**, f_n all differentiable, is f differentiable?
STILL NO: Take

```
> f_n := abs(x)^(1+(1/n));
```

$$f\_n := |x|^{1+\frac{1}{n}} \quad \textbf{(3)}$$

Limit is absolute value function f(x) = |x|:

```
> plot([seq(abs(x)^(1+(1/(2*n))),n=1..5)],x=-1..1);
```



Problem is that derivatives do not converge uniformly:

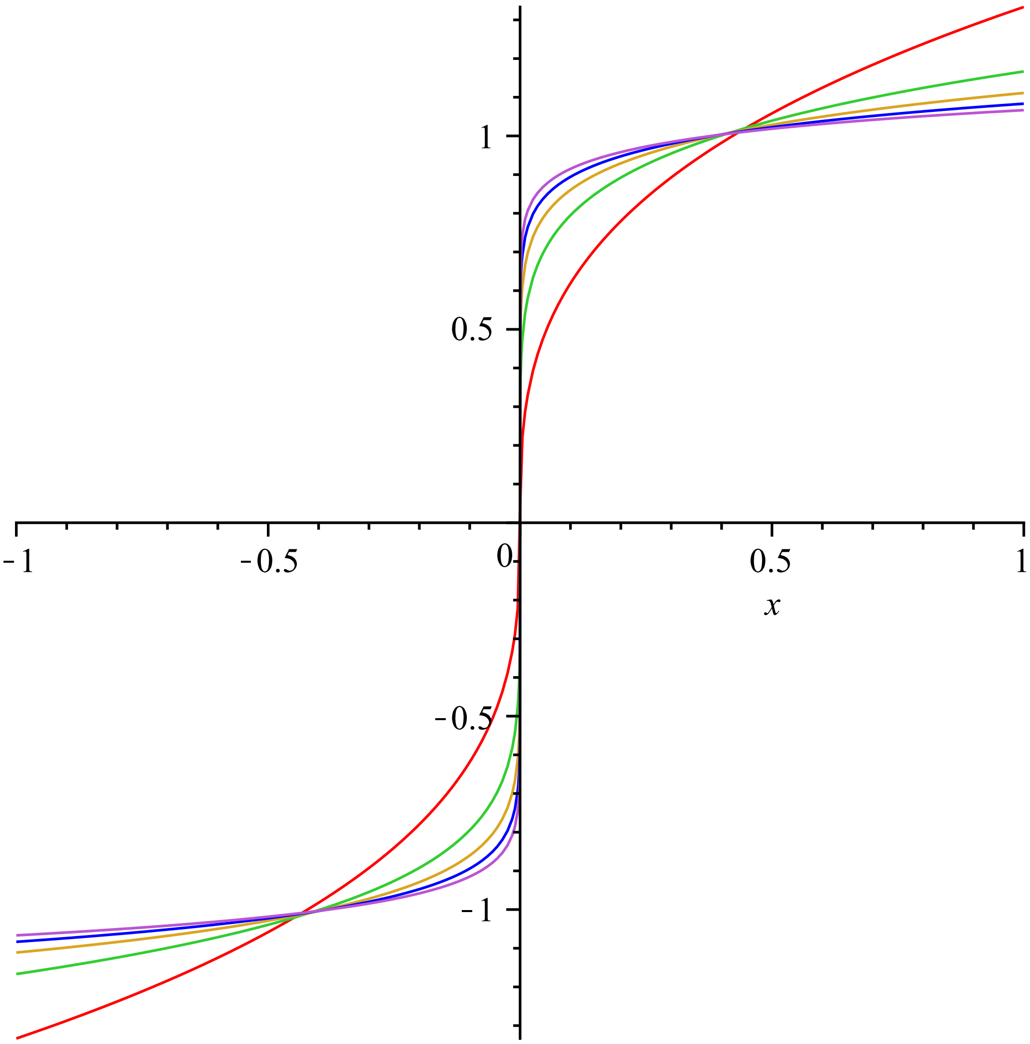
```
> fprime_n := simplify(diff(f_n,x));
```

$$fprime\_n := \frac{(n+1)\operatorname{abs}(1,x)\left|\frac{|x|^{\frac{n+1}{n}}}{x}\right|}{n} \quad \textbf{(4)}$$

```
> fprimelist := map(proc(k) subs(n=k,fprime_n) end proc, [3,6,9,12,
  15]);
```

$$fprimelist := \left[\frac{4}{3}\operatorname{abs}(1,x)\left|\frac{|x|^{4/3}}{x}\right|, \frac{7}{6}\operatorname{abs}(1,x)\left|\frac{|x|^{7/6}}{x}\right|, \frac{10}{9}\operatorname{abs}(1,x)\left|\frac{|x|^{10/9}}{x}\right|, \frac{13}{12}\operatorname{abs}(1,x)\left|\frac{|x|^{13/12}}{x}\right|, \frac{16}{15}\operatorname{abs}(1,x)\left|\frac{|x|^{16/15}}{x}\right|\right] \quad \textbf{(5)}$$

```
> plot(fprimelist,x=-1..1);
```



1
0.5
-1
-0.5
0
0.5
1
x
-0.5
-1