Sample outline for problem set 01
Math 131A

Definitions: The definitions you should copy from the text are: (Sec. 3, starting Defn. 3.3) absolute value, distance between a and b. (Sec. 4) maximum, minimum, closed interval, open interval, half-open interval, upper bound, bounded above, lower bound, bounded below, bounded, supremum, infimum. Note that you do not need to copy theorems from the text for the outline; in fact, I prefer that you don't, though I won't take off points for it.

Below: **A** = Assume, **C** = Conclude, **ABC** = Assume By way of Contradiction.

1. **A**. $a, b, c \in \mathbf{R}$.
A. $a \leq b$ and $b < c$ (i.e., $b \leq c$ and $b \neq c$).
(stuff)
C. $a < c$ (i.e., $a \leq c$ and $a \neq c$).

2. **A**. For all $x < a$, we have $x \leq b$.
ABC. $b < a$.
(stuff)
Contradiction.

3. Goal: For each of sets (e), (h), (k), (l) (the letter ℓ), (v), from Ex. 4.1, list 3 different upper bounds and the sup of the set, or write NOT BOUNDED ABOVE.

4. Guess: $\sup S =??$ (say U)

Part 1: WTS U is an upper bound for S.
A. $x \in S$.
(stuff)
C. $x \leq U$.

Part 2: WTS U is the least upper bound of S. I.e., we WTS that if $v < U$, then v is not an upper bound for S.
A. $v < U$.
(stuff)
So there exists some $x \in S$ such that $x > v$.
C. v is not an upper bound for S.

5. Guess: $\inf S =??$ (say L)

Part 1: WTS L is a lower bound for S.
A. $x \in S$.
(stuff)
C. $x \geq L$.

Part 2: WTS L is the greatest lower bound of S. I.e., we WTS that if $L < \ell$, then ℓ is not a lower bound for S.
A. $L < \ell$.
(stuff)
So there exists some $x \in S$ such that $x < \ell$.
C. ℓ is not a lower bound for S.

6. Guess: $\sup S = ??$ (say U)

Part 1: WTS U is an upper bound for S.
A. $x \in S$.
(stuff)
C. $x \leq U$.

Part 2: WTS U is the least upper bound of S. I.e., we WTS that if $v < U$, then v is not an upper bound for S.
A. $v < U$.
(stuff)
So there exists some $x \in S$ such that $x > v$.
C. v is not an upper bound for S.

(Note that the outlines for 4 and 6 are basically the same.)

7. Negations are:

- There exists $\epsilon > 0$, such that for every $s \in S$, we have $u - s \geq \epsilon$.
- $u \neq \sup S$.

A. There exists $\epsilon > 0$, such that for every $s \in S$, we have $u - s \geq \epsilon$.
(stuff)
C. $u \neq \sup S$.

Converse switches **A**. and **C**.