Sample Exam 1
Math 131A, Spring 2024

1. (10 points) Let S be a nonempty subset of $\mathbf{R}$, and let u be an upper bound for S. State the Arbitrarily Close Criterion for u and S.

2. (12 points) Suppose a_n, b_n, and c_n are sequences of positive real numbers such that $\lim a_n = 2$, $\lim b_n = 3$, and $\lim c_n = 5$. Determine the value of

$$\lim \frac{7c_n}{\sqrt{a_n b_n + c_n}},$$

and carefully use the limit laws to justify (prove) your answer. (In particular, each time you use a limit law, state which limit law you are using.)

In questions 3–5, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) **TRUE/FALSE:** Let S be a nonempty subset of $\mathbf{R}$ such that $3 \in S$ and $\sup S = 7$. Then it must be the case that $5 \in S$.

4. (12 points) **TRUE/FALSE:** Let S be a nonempty bounded subset of $\mathbf{R}$. It is possible that $s \geq 4$ for all $s \in S$, that S has no lower bounds ℓ such that $\ell > 4$, and that $4 \notin S$.

5. (12 points) **TRUE/FALSE:** Let a_n be a sequence such that $|a_n| \leq 3$ for all n. Then it must be the case that a_n converges.

6. (14 points) **PROOF QUESTION.** Use the definition of limit to prove that

$$\lim \frac{n+11}{2n+7} = \frac{1}{2}.$$

7. (14 points) **PROOF QUESTION.** Let

$$S = \left\{ x \in \mathbf{Q} \mid x > \sqrt{5} \right\}.$$

Prove that $\inf S = \sqrt{5}$.

8. (14 points) **PROOF QUESTION.** Let s_n be a real-valued sequence.

(a) Define what it means to say that $\lim s_n = 5$.

(b) Use the definition of limit to prove that if $\lim s_n = 5$, then

$$\lim 3s_n = 15.$$

(Do **NOT** use the limit laws in your proof; in fact, the goal of this problem is to prove a special case of the limit laws.)