Format and topics for final exam
Math 131A

General information. The final will be a little less than twice as long as our in-class exams, with 135 minutes in which to complete it. It will take place in our usual room.

The final will be **cumulative**; in other words, the final will cover the topics on this sheet and also on the previous three review sheets. However, the exam will somehwat emphasize the material listed here from the integration notes, the notes on uniform vs. pointwise convergence, and Sections 34 and 23–26. As always, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs may help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

As usual: No books or notes allowed, and four basic types of questions, namely, computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

Integration 1	partition	ith subinterval
	refinement	common refinement
	nth standard partition	$m(f;P,i)$, $M(f;P,i)$
	upper (resp. lower) Riemann sum	$U(f;P)$, $L(f;P)$
	upper (resp. lower) Riemann integral	$\overline{\int_a^b} f(x)\,dx$, $\underline{\int_a^b} f(x)\,dx$
	integrable	Riemann integral
	$\int_a^b f(x)\,dx$	
Sect. 34	$\int_a^b f(x)\,dx$ $(a > b)$	indefinite integral
Sect. 23	power series	coefficients
	radius of convergence	interval of convergence
Sect. 24	pointwise convergence	$f_n \to f$ pointwise
	uniform convergence	$f_n \to f$ uniformly
Sect. 25	uniformly Cauchy	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. Many of the important examples we have encountered have appeared in the assigned problems. In addition, you should also know:

Integration 1: Standard partition with n subintervals; continuous functions and standard partition into n pieces. Integral of a constant function.

Sect. 23: Exmp. 1–5 (possibilities for radius of convergence), Exmp. 6 (calculating R.o.C. with skipped terms). Exmp. 8 (pointwise convergence doesn't preserve continuity).

Uniform vs. pointwise: The five NO's (pointwise convergence doesn't preserve d/dx, continuity, $\int$) from slides in class; more examples from handout.

Sect. 24: Pointwise vs. uniform convergence (Exmp. 1–6). Proving non-uniform convergence by calculating $\lim_{n\to\infty} \sup\{|f_n(x) - f(x)| \mid x \in S\}$.

Sect. 25: Series of functions (Exmp. 1–3). Converging to a continuous function does not imply uniform convergence (Exmp. 4). Uniform continuity implies terms converge to 0 (nth term test) (Exmp. 5).

Sect. 26: Using term-by-term integration and differentiation (Exmp. 1).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Integration 1: Finer partitions are better (Lem. 1.8). Lower integral is $\leq$ upper integral (Thm. 1.9). Sequential criteria for integrability (Lem. 1.10)

Integration 2: Integration laws (Thm. 2.2). Additivity of domain (Thm. 2.3). Continuity implies integrability (Thm. 2.4). Triangle inequality for integrals (Thm. 2.5).

Sect. 34 Fundamental Theorems of Calculus.

Sect. 23: Definition and computation of radius of convergence (Thm. 23.1). R.o.C. can be computed by Ratio Test (p. 172/Cor. 12.3).

Sect. 24: Uniform convergence preserves continuity (Thm. 24.3, series version Thm. 25.5). $f_n \to f$ uniformly if and only if $\lim_{n\to\infty} \sup \{|f_n(x) - f(x)| \mid x \in S\} = 0$ (Rem. 24.4).

Sect. 25: Uniform convergence can be exchanged with integration (Thm. 25.2). Uniformly Cauchy implies uniformly convergent (Thm. 25.4, series version Thm. 25.6). **Weierstrass M-test** (Thm. 25.7).

Sect. 26: Power series converge uniformly in $[-R_1, R_1]$ for $R_1 <$ R.o.C. (Thm. 26.1) and to a continuous function within R.o.C. (Thm. 26.2). Power series can be safely integrated term-by-term (Thm. 26.4) and differentiated term-by-term (Thm. 26.5).

Not on exam. (Sect. 23) Power series with center $\neq 0$, Exmp. 7. (Sect. 26) Abel's Theorem, Exmp. 2–3.

Topics that may be on the final, but not as the sole focus of a question. In other words, questions on the exam may relate to these topics, but there will not be any questions that cover *only* these topics. (Sects. 1–3) Everything.

Good luck.