Format and topics
Exam 3, Math 131A

General information. Exam 3 will be a timed test of 75 minutes, covering Sections 18, 19, 20, 28, 29, and sections 1 and 2 of the class notes on Integration (i.e., roughly, PS07, PS08, and PS09). The exam will be cumulative only to the extent that these sections rely on previous material; for example, you should still know what the supremum of a subset of $\mathbb{R}$ is. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to define the supremum of a subset of $\mathbb{R}$.

As before, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Exam 3 will follow the usual ground rules. In particular, no books, notes, or calculators are allowed, and there will be the same four types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

Sect. 18	bounded (function)	strictly increasing (function)
	strictly decreasing (function)	
Sect. 19	uniformly continuous on a set	uniformly continuous
Sect. 20	$\lim_{x\to a^S} f(x) = L$	$\lim_{x\to a} f(x) = L$
	$\lim_{x\to a^+} f(x) = L$	$\lim_{x\to a^-} f(x) = L$
	$\lim_{x\to\infty} f(x) = L$	ϵ-δ "definition" of limit
Sect. 28	differentiable at a	derivative at a
	$f'(a)$	
Sect. 29	strictly increasing/decreasing (function)	increasing/decreasing (function)
Integration 1	partition	ith subinterval
	refinement	common refinement
	nth standard partition	$m(f;P,i)$, $M(f;P,i)$
	upper (resp. lower) Riemann sum	$U(f;P)$, $L(f;P)$
	upper (resp. lower) Riemann integral	$\overline{\int_a^b} f(x)\,dx$, $\underline{\int_a^b} f(x)\,dx$
	integrable	Riemann integral
	$\int_a^b f(x)\,dx$	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 18: Applications of Intermediate Value Theorem (Exmps. 1–2).
Sect. 19: Proving a function is pointwise continuous on its domain (Exmp. 1). Uniform vs. pointwise continuity (Exmps. 2–4).
Sect. 20: Computing examples of limits through continuity (Exmps. 1–3), the definition (Exmp. 4), and composition (Exmp. 5–6). If limits of g and f exist, limit of $g \circ f$ may not exist (Exmp. 7).
Sect. 28: Computing derivatives from the definition (Exmps. 1–3), algebraic rules (Exmp. 4), chain rule (Exmp. 5).
Sect. 29: Increasing and decreasing functions (Exmp. 1).

Integration 1: Standard partition with n subintervals; continuous functions and standard partition into n pieces. Integral of a constant function.

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 18: Extreme Value Theorem (Minimum-Maximum Theorem) (Thm. 18.1). **Intermediate Value Theorem** (Thm. 18.2). Consequences of Intermediate Value Theorem: Cor. 18.3.

Sect. 19: Continuous functions on a closed and bounded interval are uniformly continuous (Thm. 19.2).

Sect. 20: Continuity = limit + value (Rem. 20.2); limit laws (Thm. 20.4); limits of compositions (Thm. 20.5). Equivalence of sequential and ϵ-δ definitions of limits (Thm. 20.6, Cors. 20.7–20.8, Discussion 20.9).

Sect. 28: Differentiability implies continuity (Thm. 28.2); algebraic laws of differentiation (Thm. 28.3); Chain Rule (Thm. 28.4).

Sect. 29: Interior Extremum Theorem (Thm. 29.1); **Mean Value Theorem** (Thm. 29.3) (special case: Rolle's Theorem (Thm. 29.2)). Zero Derivative Theorems (Cors. 29.4–29.5). **THE BOX** (Thm. 29.7).

Integration 1: Finer partitions are better (Lem. 1.8). Lower integral is $\leq$ upper integral (Thm. 1.9). Sequential criteria for integrability (Lem. 1.10)

Integration 2: Integration laws (Thm. 2.2). Additivity of domain (Thm. 2.3). Continuity implies integrability (Thm. 2.4). Triangle inequality for integrals (Thm. 2.5).

Not on exam. (Sect. 18) Thms. 18.4–18.6. (Sect. 19) pp. 144–152. (Sect. 29) Intermediate value theorem for derivatives (Thm. 29.8), differentiability of inverse functions (Thm. 29.9).

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the proof notes. You do not need to memorize information from the proof notes, but you do need to be able to apply it.

Good luck.