Format and topics
Exam 2, Math 131A

General information. Exam 2 will be a timed test of 75 minutes, covering Sections 9, 10, 11, 12, 14, 15, and 17 of the text, as well as the notes on Asymptotics. More succinctly, the exam will focus on PS04–06. The exam will be cumulative only to the extent that Sections 9–12, 14–15, 17, the notes on Asyptotics, and PS04–06 rely on previous material; for example, you should still know what the supremum of a subset of $\mathbb{R}$ is. However, there will not be any questions on the exam that only cover old material; for example, you will not be asked to define the supremum of a subset of $\mathbb{R}$.

As before, most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we've studied, you should be in good shape. You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should defintely spend time memorizing the *statements* of the important theorems in the text.

Exam 2 will follow the same ground rules as Exam 1 did. In particular, no books, notes, or calculators are allowed, and there will be the same four types of questions: computations, statements of definitions and theorems, proofs, and true/false with justification.

Definitions. The most important definitions we have covered are:

Sect. 9	bounded (sequence)	$\lim s_n = +\infty$
	$\lim s_n = -\infty$	diverge to $\pm\infty$
Asymptotics	$a_n << b_n$	
Sect. 10	increasing sequence	decreasing sequence
	monotone sequence	monotonic sequence
	strictly increasing	strictly decreasing
	$\limsup s_n$	$\liminf s_n$
	Cauchy sequence	
Sect. 11	subsequence	subsequential limit
	k-tail	
Sect. 14	sequence of partial sums	converge (series)
	diverge (series)	diverge to $\pm\infty$
	converge absolutely	geometric series
Sect. 15	nonincreasing function	alternating series
Sect. 17	continuous at a point	continuous on a set
	continuous	ϵ-δ continuity
	discontinuous	composition $f \circ g$
	$(f+g)(x)$, $(fg)(x)$, $(f/g)(x)$	

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Sect. 9: $(-1)^n$ is bounded but divergent. Using limit laws (Exs. 1–3). Infinite limits (Exs. 4–7).
Sect. 11: Subsequences and subsequential limits (Exmps. 1–4, 5–8, 9–12)
Sect. 14: Geometric series (Exmp. 1), p-series (Exmp. 2). Applying convergence tests (Exmps. 3–9).
Sect. 15: p-series (Exmps. 1–2).
Sect. 17: Proving functions continuous and discontinuous (Exmps. 1–3, 4–5).

Theorems, results, algorithms. The most important theorems, results, and algorithms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name, as listed below in **boldface**, or a vague description).

Sect. 9: Limit laws: Convergent $\Rightarrow$ bounded (Thm. 9.1), constant multiple (Thm. 9.2), sum (Thm. 9.3), product (Thm. 9.4), quotient (Thm. 9.6). Basics: $\lim(1/n^p)$ $(p > 0)$, $\lim a^n$ $(|a| < 1)$, $\lim n^{1/n}$, $\lim a^{1/n}$ $(a > 0)$, (Thm. 9.7). $\lim s_n = +\infty$: product, reciprocal (Thms. 9.9, 9.10).

Asymptotics: Exers. 9.12, 9.14, 9.15. **The Asymptotics Theorem**: $1 << \ln n << n^t << a^n << n!$.

Sect. 10: Monotone Convergence Theorem (Thm. 10.2), unbounded MCT (Thm. 10.4, Cor. 10.5). Limit vs. $\limsup$, $\liminf$ (Thm. 10.7). **Cauchy Convergence Theorem** (Thm. 10.11).

Sect. 11: Subsequence of convergent sequence converges (Thm. 11.2). Every subsequence has a monotone subsequence ("eyeballs", Thm. 11.3). Every sequence has a subsequence converging to inf (resp. sup) (Cor. 11.4). **Bolzano-Weierstrass Theorem** (Thm. 11.5).

Sect. 14: Cauchy Criterion for Convergence (Thm. 14.4). Divergence Test/nth term test (Thm. 14.5). Comparison Test (Thm. 14.6); absolute convergence implies convergence (Cor. 14.7). Ratio Test (Thm. 14.8), Root Test (Thm. 14.9).

Notes on series: Limit Comparison Test, simplified Root Test and Ratio Test.

Sect. 15: p-series (Thm. 15.1), Integral Tests (15.2). Alternating series (Thm. 15.3).

Sect. 17: Equivalence of definitions of continuity (Thm. 17.2). Continuity laws (Thms. 17.3–17.5).

Not on exam. (Sect. 10) Discussion of decimals. (Sect. 11) Thms. 11.7–11.8, closed set. (Sect. 12) Specific results on lim inf and lim sup. (Notes on rearrangements) All.

Other. You should have a working familiarity with the techniques and strategies for proof and logic tips from the proof notes. You do not need to memorize information from the proof notes, but you do need to be able to apply it.

Good luck.